Pressure And Head

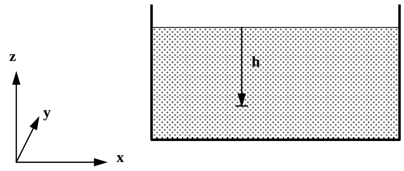
We have the vertical pressure relationship

$$\frac{dp}{dz} = -\rho g,$$

integrating gives

$$p = -\rho g z + constant$$

measuring z from the free surface so that $z = -h$



$$p = \rho g h + \text{constant}$$

surface pressure is atmospheric, $p_{\text{atmospheric}}$.

$$p_{\text{atmospheric}} = \text{constant}$$

so

$$p = \rho g h + p_{\text{atmospheric}}$$

It is convenient to take atmospheric pressure as the datum

Pressure quoted in this way is known as gauge pressure i.e.

Gauge pressure is

$$p_{gauge} = \rho g h$$

The lower limit of any pressure is the pressure in a perfect vacuum.

Pressure measured above a perfect vacuum (zero) is known as absolute pressure

Absolute pressure is

$$p_{absolute} = \rho g h + p_{atmospheric}$$

Absolute pressure = Gauge pressure + Atmospheric

A gauge pressure can be given using height of *any* fluid.

$$p = \rho g h$$

This vertical height is the *head*.

If pressure is quoted in *head*, the density of the fluid *must* also be given.

Example:

What is a pressure of 500 kNm^{-2} in head of water of density, $\rho = 1000$ kgm^{-3}

Use $p = \rho g h$,

$$h = \frac{p}{\rho g} = \frac{500 \times 10^3}{1000 \times 9.81} = 50.95m \text{ of water}$$

In head of Mercury density $\rho = 13.6 \times 10^3$ kgm^{-3}.

$$h = \frac{500 \times 10^3}{13.6 \times 10^3 \times 9.81} = 3.75m \text{ of Mercury}$$

In head of a fluid with relative density $\gamma = 8.7$.

(remember $\rho = \gamma \times \rho_{water}$)

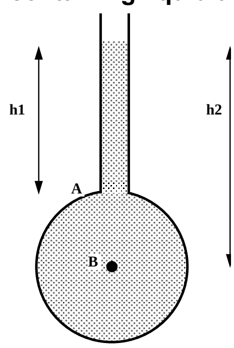
$$h = \frac{500 \times 10^3}{(8.7 \times 1000) \times 9.81} = 5.86m \text{ of fluid } \gamma = 8.7$$

Pressure Measurement By Manometer

Manometers use the relationship between pressure and head to measure pressure

The Piezometer Tube Manometer

The simplest manometer is an open tube.
This is attached to the top of a container with liquid at pressure. containing liquid at a pressure.



The tube is open to the atmosphere,

The pressure measured is relative to atmospheric so it measures *gauge pressure*.

Pressure at A = pressure due to column of liquid h_1

$$p_A = \rho g h_1$$

Pressure at B = pressure due to column of liquid h_2

$$p_B = \rho g h_2$$

Problems with the Piezometer:

1. Can only be used for liquids

2. Pressure must above atmospheric

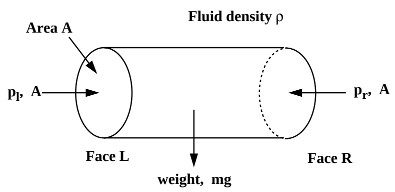
3. Liquid height must be convenient
   i.e. not be too small or too large.

An Example of a Piezometer.

What is the maximum gauge pressure of water that can be measured by a Piezometer of height 1.5m?

And if the liquid had a relative density of 8.5 what would the maximum measurable gauge pressure?

Equality Of Pressure At The Same Level In A Static Fluid



Horizontal cylindrical element

cross sectional area = A

mass density = ρ

left end pressure = p_l

right end pressure = p_r

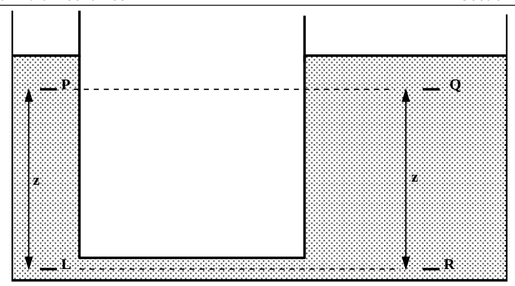
For equilibrium the sum of the forces in the x direction is zero.

$$p_l A = p_r A$$

$$p_l = p_r$$

Pressure in the horizontal direction is constant.

This true for any *continuous* fluid.



We have shown

$$p_l = p_r$$

For a vertical pressure change we have

$$p_l = p_p + \rho g z$$

and

$$p_r = p_q + \rho g z$$

so

$$p_p + \rho g z = p_q + \rho g z$$

$$p_p = p_q$$

Pressure at the two equal levels are the same.

The "U"-Tube Manometer

"U"-Tube enables the pressure of both liquids and gases to be measured

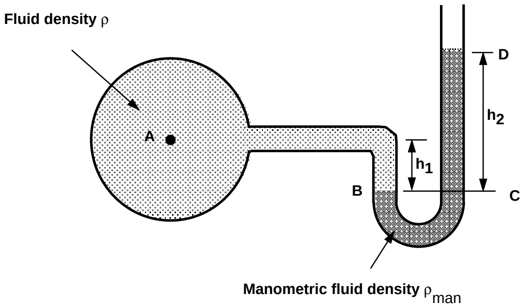
"U" is connected as shown and filled with *manometric fluid*.

Important points:

1. **The manometric fluid density should be greater than of the fluid measured.**

   $\rho_{man} > \rho$

2. **The two fluids should not be able to mix they must be immiscible.**



We know:

Pressure in a continuous static fluid is the same at any horizontal level.

pressure at B = pressure at C

$$p_B = p_C$$

For the left hand arm

pressure at B = pressure at A + pressure of height of liquid being measured

$$p_B = p_A + \rho g h_1$$

For the right hand arm

pressure at C = pressure at D + pressure of height of manometric liquid

$$p_C = p_{atmospheric} + \rho_{man} g h_2$$

We are measuring *gauge pressure* we can subtract $p_{atmospheric}$ giving

$$p_B = p_C$$

$$p_A = \rho_{man} g h_2 - \rho g h_1$$

What if the fluid is a gas?

Nothing changes.

The manometer work exactly the same.

BUT:

As the manometric fluid is liquid (usually mercury , oil or water)

And Liquid density is much greater than gas,

$$\rho_{man} >> \rho$$

$\rho g h_1$ can be neglected,

and the gauge pressure given by

$$p_A = \rho_{man} g h_2$$

An example of the U-Tube manometer.

Using a u-tube manometer to measure gauge pressure of fluid density ρ = 700 kg/m^3, and the manometric fluid is mercury, with a relative density of 13.6.

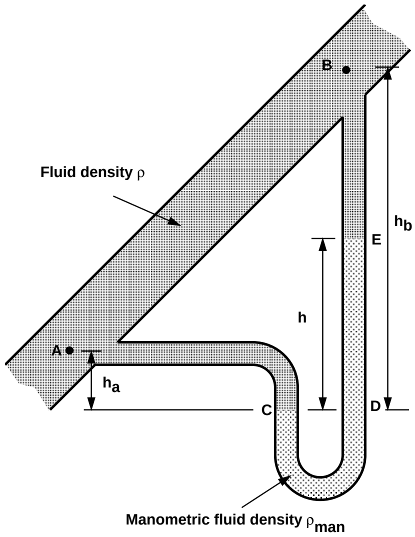
What is the gauge pressure if:

a) h_1 = 0.4m and h_2 = 0.9m?

b) h_1 stayed the same but h_2 = -0.1m?

Pressure difference measurement Using a "U"-Tube Manometer.

The "U"-tube manometer can be connected at both ends to measure *pressure difference* between these two points



pressure at C = pressure at D

$$p_C = p_D$$

$$p_C = p_A + \rho g h_a$$

$$p_D = p_B + \rho g (h_b + h) + \rho_{man} g h$$

$$p_A + \rho g h_a = p_B + \rho g (h_b + h) + \rho_{man} g h$$

Giving the pressure difference

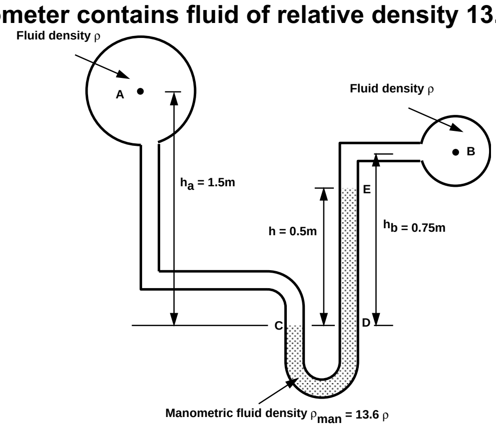
$$p_A - p_B = \rho g (h_b - h_a) + (\rho_{man} - \rho) g h$$

Again if the fluid is a gas $\rho_{man} >> \rho$, then the terms involving ρ can be neglected,

$$p_A - p_B = \rho_{man} g h$$

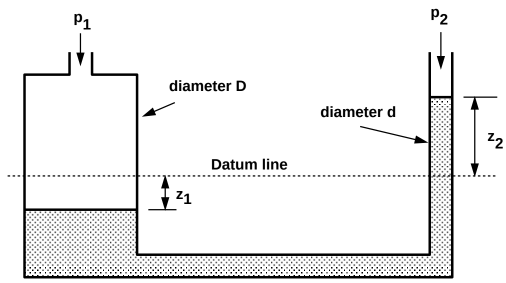
An example using the u-tube for pressure difference measuring

In the figure below two pipes containing the same fluid of density ρ = 990 kg/m^3 are connected using a u-tube manometer.
What is the pressure between the two pipes if the manometer contains fluid of relative density 13.6?



Advances to the "U" tube manometer

Problem: Two reading are required.
Solution: Increase cross-sectional area of one side.
Result: One level moves much more than the other.



If the manometer is measuring the pressure difference of a gas of *(p_1 - p_2)* as shown, we know

$$p_1 - p_2 = \rho_{man} g h$$

volume of liquid moved from the left side to the right

$$= z_2 \times (\pi d^2 / 4)$$

The fall in level of the left side is

$$z_1 = \frac{\text{Volume moved}}{\text{Area of left side}}$$
$$= \frac{z_2\left(\pi d^2 / 4\right)}{\pi D^2 / 4}$$
$$= z_2\left(\frac{d}{D}\right)^2$$

Putting this in the equation,

$$p_1 - p_2 = \rho g\left[z_2 + z_2\left(\frac{d}{D}\right)^2\right]$$
$$= \rho g z_2\left[1 + \left(\frac{d}{D}\right)^2\right]$$

If D >> d then $(d/D)^2$ is very small so

$$p_1 - p_2 = \rho g z_2$$

Problem: Small pressure difference, movement cannot be read.

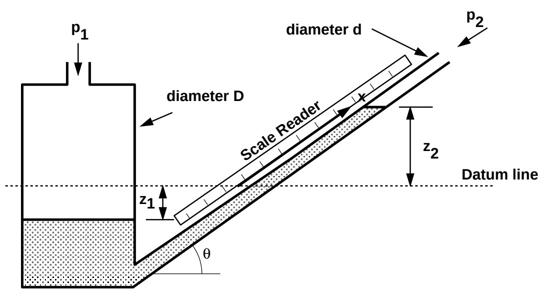
Solution 1: Reduce density of manometric fluid.

Result: Greater height change - easier to read.

Solution 2: Tilt one arm of the manometer.

Result: Same height change - but larger movement along the manometer arm - easier to read.

<u>Inclined manometer</u>



The pressure difference is still given by the height change of the manometric fluid.

$$p_1 - p_2 = \rho g z_2$$

but,

$$z_2 = x \sin\theta$$
$$p_1 - p_2 = \rho g x \sin\theta$$

The sensitivity to pressure change can be increased further by a greater inclination.

<u>Example of an inclined manometer.</u>

An inclined manometer is required to measure an air pressure of 3mm of water to an accuracy of +/- 3%. The inclined arm is 8mm in diameter and the larger arm has a diameter of 24mm. The manometric fluid has density ρ_{man} = 740 kg/m^3 and the scale may be read to +/- 0.5mm.

What is the angle required to ensure the desired accuracy may be achieved?

Choice Of Manometer

Take care when fixing the manometer to vessel
Burrs cause local pressure variations.

Disadvantages:

- **Slow response - only really useful for very slowly varying pressures - no use at all for fluctuating pressures;**
- **For the "U" tube manometer two measurements must be taken simultaneously to get the h value.**
- **It is often difficult to measure small variations in pressure.**
- **It cannot be used for very large pressures unless several manometers are connected in series;**
- **For very accurate work the temperature and relationship between temperature and ρ must be known;**

Advantages of manometers:

- **They are very simple.**
- **No calibration is required - the pressure can be calculated from first principles.**

Forces on Submerged Surfaces in Static Fluids

We have seen these features of static fluids

- **Hydrostatic vertical pressure distribution**
- **Pressures at any equal depths in a continuous fluid are equal**
- **Pressure at a point acts equally in all directions (Pascal's law).**
- **Forces from a fluid on a boundary acts at right angles to that boundary.**

Fluid pressure on a surface

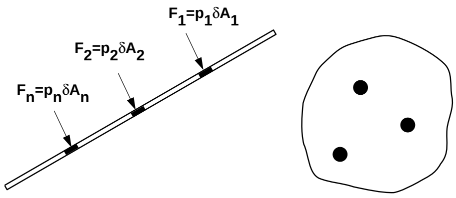
Pressure is force per unit area.

Pressure *p* acting on a small area δA exerted force will be

$$F = p \times \delta A$$

Since the fluid is at rest the force will act at right-angles to the surface.

General submerged plane



The total or *resultant* force, *R*, on the plane is the sum of the forces on the small elements i.e.

$$R = p_1\delta A_1 + p_2\delta A_2 + \ldots\ldots p_n\delta A_n = \sum p\delta A$$

and

This resultant force will act through the *centre of pressure*.

For a plane surface all forces acting can be represented by one single *resultant force,* acting at right-angles to the plane through the *centre of pressure*.

Horizontal submerged plane

The pressure, *p*, will be equal at all points of the surface.

The resultant force will be given by

$$R = \text{pressure} \times \text{area of plane}$$

$$R = pA$$

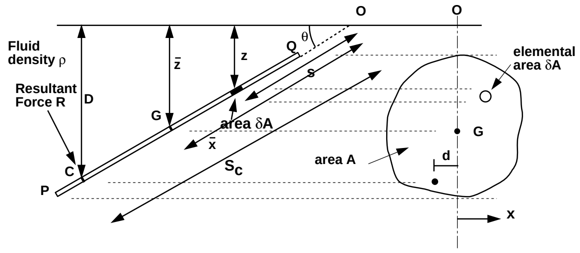
Curved submerged surface

Each elemental force is a different magnitude and in a different direction (but still normal to the surface.).

It is, in general, not easy to calculate the resultant force for a curved surface by combining all elemental forces.

The sum of all the forces on each element will always **be less than the sum of the individual forces, $\sum p\delta A$.**

Resultant Force and Centre of Pressure on a general plane surface in a liquid.



Take pressure as zero at the surface.

Measuring down from the surface, the pressure on an element δA, depth z,

$$p = \rho g z$$

So force on element

$$F = \rho g z \delta A$$

Resultant force on plane

$$R = \rho g \sum z \delta A$$

(assuming ρ and *g* as constant).

$\sum z\delta A$ is known as

the *1st Moment of Area* of the plane PQ about the free surface.

And it is known that

$$\sum z\delta A = A\bar{z}$$

A is the area of the plane

$\bar{z}$ is the distance to the *centre of gravity (centroid)*

In terms of distance from point O

$$\sum z\delta A = A\bar{x}\sin\theta$$

= 1st moment of area × *sinθ*
about a line through O

(as $\bar{z} = \bar{x}\sin\theta$)

The resultant force on a plane

$$R = \rho g A\bar{z}$$
$$= \rho g A\bar{x}\sin\theta$$

This resultant force acts at right angles through the centre of pressure, C, at a depth D.

How do we find this position?

Take moments of the forces.

As the plane is in equilibrium:
The moment of *R* will be equal to the sum of the moments of the forces on all the elements δA about the same point.

It is convenient to take moment about O

The force on each elemental area:

$$\text{Force on } \delta A = \rho g z \delta A$$
$$= \rho g\, s \sin\theta\, \delta A$$

the moment of this force is:

$$\text{Moment of Force on } \delta A \text{ about O} = \rho g\, s\sin\theta\,\delta A \times s$$
$$= \rho g \sin\theta\,\delta A s^2$$

ρ , g and θ are the same for each element, giving the total moment as

Sum of moments $= \rho g \sin\theta \sum s^2 \delta A$

Moment of R about O = $R \times S_c = \rho g A\bar{x}\sin\theta\, S_c$

Equating

$$\rho g A\bar{x}\sin\theta\, S_c = \rho g \sin\theta \sum s^2\delta A$$

The position of the centre of pressure along the plane measure from the point O is:

$$S_c = \frac{\sum s^2 \delta A}{A\bar{x}}$$

How do we work out the summation term?

This term is known as the
2nd Moment of Area , I_o,
of the plane
(about the axis through O)

$$\text{2nd moment of area about O} = I_o = \sum s^2 \delta A$$

It can be easily calculated for many common shapes.

The position of the *centre of pressure* along the plane measure from the point O is:

$$S_c = \frac{2^{\text{nd}} \text{ Moment of area about a line through O}}{1^{\text{st}} \text{ Moment of area about a line through O}}$$

and

Depth to the *centre of pressure* is

$$D = S_c \sin\theta$$

How do you calculate the 2^{nd} moment of area?

2^{nd} moment of area is a geometric property.

It can be found from tables - BUT only for moments about an axis through its centroid = I_{GG}.

Usually we want the 2^{nd} moment of area about a different axis.

Through O in the above examples.

We can use the *parallel axis theorem* to give us what we want.

The *parallel axis theorem* can be written

$$I_o = I_{GG} + A\bar{x}^2$$

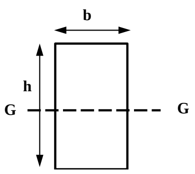
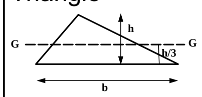
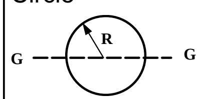
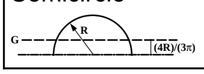
We then get the following equation for the position of the centre of pressure

$$S_c = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

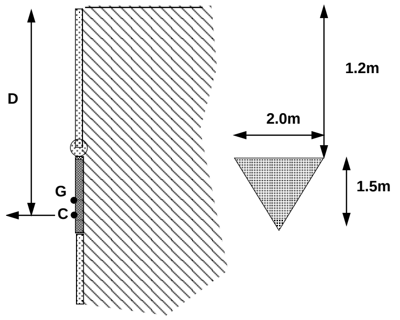
$$D = \sin\theta\left(\frac{I_{GG}}{A\bar{x}} + \bar{x}\right)$$

(In the examination the parallel axis theorem and the I_{GG} will be given)

The 2^{nd} moment of area about a line through the centroid of some common shapes.

Shape	Area A	2^{nd} moment of area, I_{GG}, about an axis through the centroid
Rectangle 	bd	$\frac{bd^3}{12}$
Triangle 	$\frac{bd}{2}$	$\frac{bd^3}{36}$
Circle 	πR^2	$\frac{\pi R^4}{4}$
Semicircle 	$\frac{\pi R^2}{2}$	$0.1102 R^4$

An example:
Find the moment required to keep this triangular gate closed on a tank which holds water.



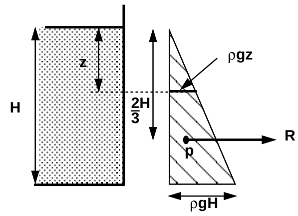
Submerged vertical surface - Pressure diagrams

For vertical walls of constant width it is possible to find the resultant force and centre of pressure graphically using a *pressure diagram*.

We know the relationship between pressure and depth:

$$p = \rho g z$$

So we can draw the diagram below:



This is know as a *pressure diagram*.

Pressure increases from zero at the surface linearly by $p = \rho g z$, to a maximum at the base of $p = \rho g H$.

The *area* of this triangle represents the *resultant force per unit width* on the vertical wall,

Units of this are Newtons per metre.

$$\begin{aligned}\text{Area } &= \frac{1}{2} \times AB \times BC \\ &= \frac{1}{2} H \rho g H \\ &= \frac{1}{2} \rho g H^2\end{aligned}$$

Resultant force per unit width

$$R = \frac{1}{2} \rho g H^2 \qquad (N/m)$$

The force acts through the *centroid of the pressure diagram*.

For a triangle the centroid is at 2/3 its height i.e. the resultant force acts horizontally through the point $z = \frac{2}{3} H$.

For a vertical plane the depth to the centre of pressure is given by

$$D = \frac{2}{3} H$$

Check this against the moment method:

The resultant force is given by:

$$R = \rho g A \bar{z} = \rho g A \bar{x} \sin\theta$$

$$= \rho g \left(H \times 1 \right) \frac{H}{2} \sin\theta$$

$$= \frac{1}{2} \rho g H^2$$

and the depth to the centre of pressure by:

$$D = \sin\theta \left(\frac{I_o}{A\bar{x}} \right)$$

and by the parallel axis theorem (with width of 1)

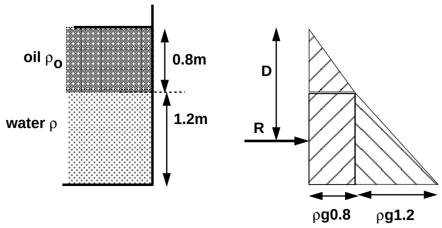
$$I_o = I_{GG} + A\bar{x}^2$$

$$= \frac{1 \times H^3}{12} + 1 \times H \left(\frac{H}{2} \right)^2 = \frac{H^3}{3}$$

Depth to the centre of pressure

$$D = \left(\frac{H^3/3}{H^2/2} \right) = \frac{2}{3} H$$

The same technique can be used with combinations of liquids are held in tanks (e.g. oil floating on water). For example:



Find the position and magnitude of the resultant force on this vertical wall of a tank which has oil floating on water as shown.

Submerged Curved Surface

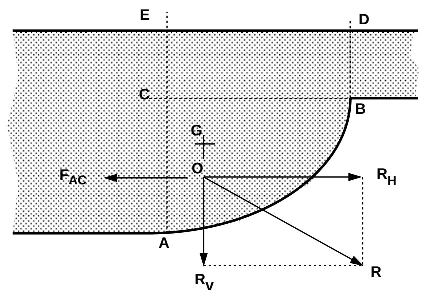
If the surface is curved the resultant force must be found by combining the elemental forces using some vectorial method.

Calculate the *horizontal and vertical components.*

Combine these to obtain the resultant force and direction.

(Although this can be done for all three dimensions we will only look at one vertical plane)

In the diagram below liquid is resting on top of a curved base.

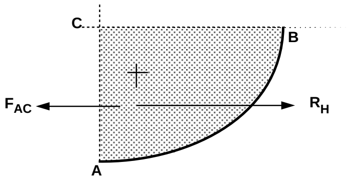


The fluid is at rest – in equilibrium.

So any element of fluid such as ABC is also in equilibrium.

Consider the Horizontal forces

The sum of the horizontal forces is zero.



No horizontal force on CB as there are no shear forces in a static fluid

Horizontal forces act only on the faces AC and AB as shown.

***F_{AC}*, must be equal and opposite to *R_H*.**

AC is the projection of the curved surface AB onto a vertical plane.

The resultant horizontal force of a fluid above a curved surface is:

R_H = Resultant force on the projection of the curved surface onto a vertical plane.

We know

1. The force on a vertical plane must act horizontally (as it acts normal to the plane).

2. That R_H must act through the same point.

So:

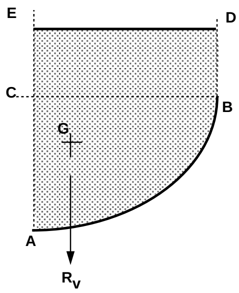
R_H acts horizontally through the centre of pressure of the projection of the curved surface onto an vertical plane.

We have seen earlier how to calculate resultant forces and point of action.

Hence we can calculate the resultant horizontal force on a curved surface.

Consider the Vertical forces

The sum of the vertical forces is zero.



There are no shear force on the vertical edges, so the vertical component can only be due to the *weight of the fluid*.

So we can say

The resultant vertical force of a fluid above a curved surface is:

R_V = Weight of fluid directly above the curved surface.

It will act vertically down through the centre of gravity of the mass of fluid.

Resultant force

The overall resultant force is found by combining the vertical and horizontal components vectorialy,

Resultant force

$$R = \sqrt{R_H^2 + R_V^2}$$

And acts through O at an angle of θ.

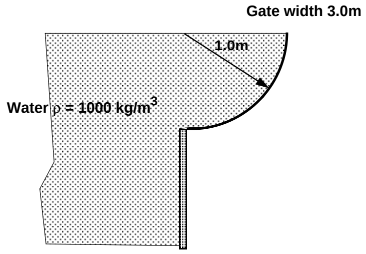
The angle the resultant force makes to the horizontal is

$$\theta = \tan^{-1}\left(\frac{R_V}{R_H}\right)$$

The position of O is the point of interaction of the horizontal line of action of R_H and the vertical line of action of R_V.

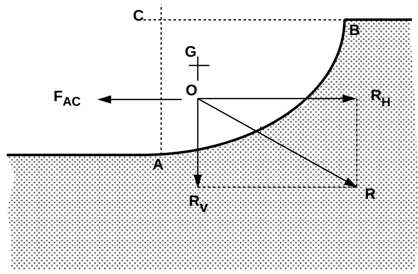
A typical example application of this is the determination of the forces on dam walls or curved sluice gates.

Find the magnitude and direction of the resultant force of water on a quadrant gate as shown below.



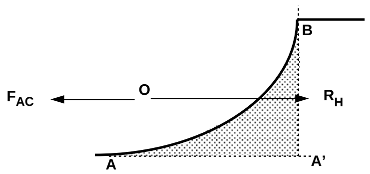
<u>What are the forces if the fluid is</u> <u>*below*</u> ***<u>the curved surface</u>*****<u>?</u>**

This situation may occur or a curved sluice gate.



The force calculation is very similar to when the fluid is above.

<u>Horizontal force</u>

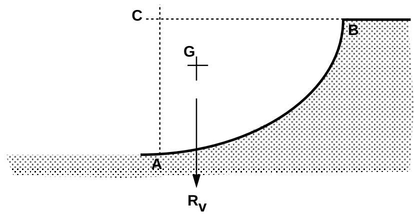


The two horizontal on the element are:
The horizontal reaction force R_H
The force on the vertical plane A'B.

The resultant horizontal force, R_H acts as shown in the diagram. Thus we can say:

The resultant horizontal force of a fluid below a curved surface is:
R_H = Resultant force on the projection of the curved surface onto a vertical plane.

<u>Vertical force</u>



What vertical force would keep this in equilibrium?

If the region above the curve were all water there would be equilibrium.

Hence: the force exerted by this amount of fluid must equal he resultant force.

The resultant vertical force of a fluid below a curved surface is:
R_v =Weight of the *imaginary* volume of fluid vertically above the curved surface.

The resultant force and direction of application are calculated in the same way as for fluids above the surface:

Resultant force

$$R = \sqrt{R_H^2 + R_V^2}$$

And acts through O at an angle of θ.

The angle the resultant force makes to the horizontal is

$$\theta = \tan^{-1}\left(\frac{R_V}{R_H}\right)$$

An example of a curved sluice gate which experiences force from fluid below.

A 1.5m long cylinder lies as shown in the figure, holding back oil of relative density 0.8. If the cylinder has a mass of 2250 kg find

a) the reaction at A b) the reaction at B