CIVE1400: An Introduction to Fluid Mechanics

Unit 2: Statics

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Module web site: **www.efm.leeds.ac.uk/CIVE/FluidsLevel1**

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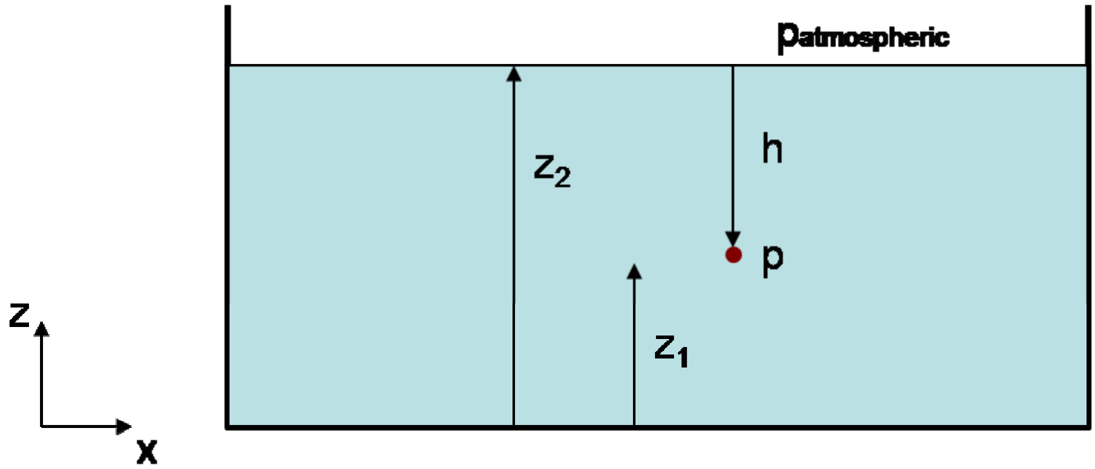
Statics : Pressure



Hydrostatic Pressure:
linear change in pressure with depth

$$p_2 - p_1 = -\rho g \left(z_2 - z_1 \right)$$

Measure depth, h, from free surface

$$p = \rho g h + p_{\text{atmospheric}}$$

Absolute pressure

$$p_{absolute} = \rho\, g\, h + p_{atmospheric}$$

Gauge pressure

$$p_{gauge} = \rho\, g\, h$$

Pressure Head

The lower limit of any pressure is the pressure in a ____________________

Pressure measured above a perfect vacuum (zero) is known as ______________ pressure

Pressure measured relative to atmospheric pressure is known as ______________ pressure

A gauge pressure can be given using height of *any* fluid.

$$p = \rho g h$$

This height, *h*, is referred to as the _______

If pressure is quoted in *head*, the ____________of the fluid ______ also be given.

Examples of pressure head calculations:

What is a pressure of 500 kNm^{-2} in head of water of density, ρ = 1000 kgm^{-3}

Use $p = \rho gh$,

$$h =$$

In head of Mercury density $\rho = 13.6\times10^3$ kgm^{-3}.

$$h =$$

In head of a fluid with relative density γ = 8.7.

(remember $\rho = \gamma \times \rho_{water}$)

$$h =$$

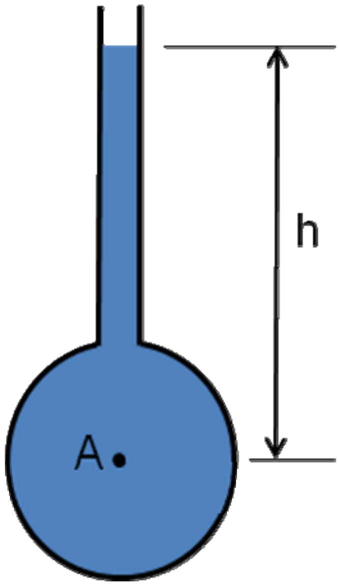
Manometers for Measuring Pressure

Manometers use the relationship between ______________ and ________ to measure pressure

Piezometer Tube

A simple open tube attached to the top of a container with liquid at pressure.

Liquid rises to a height, h, equal to the pressure in the container.



$$p_A = \rho g h$$

The pressure measured is relative to ____________ so it measures ___________ *pressure*.

Problems with the Piezometer:

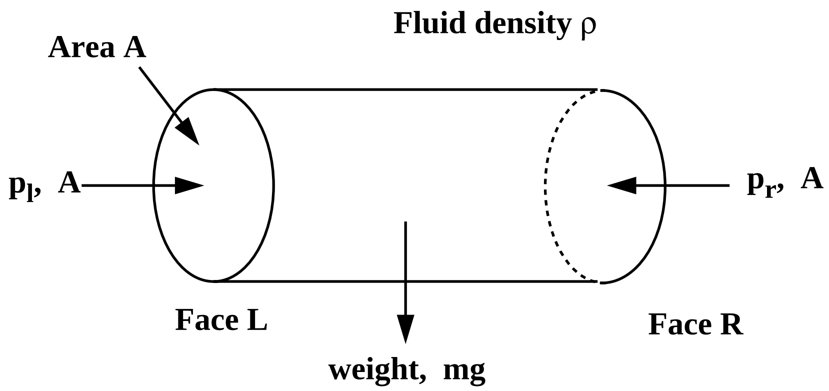
1.
2.
3.

An Examples of a Piezometer.

1. **What is the maximum gauge pressure of water that can be measured by a Piezometer of height 1.5m?**

2. **If the liquid had a relative density of 8.5 what would the maximum measurable gauge pressure?**

Equality of Pressure At The Same Level In A Static Fluid



Horizontal cylindrical element

cross sectional area = A

mass density = ρ

left end pressure = p_l

right end pressure = p_r

For equilibrium the sum of the forces in the x direction is zero.

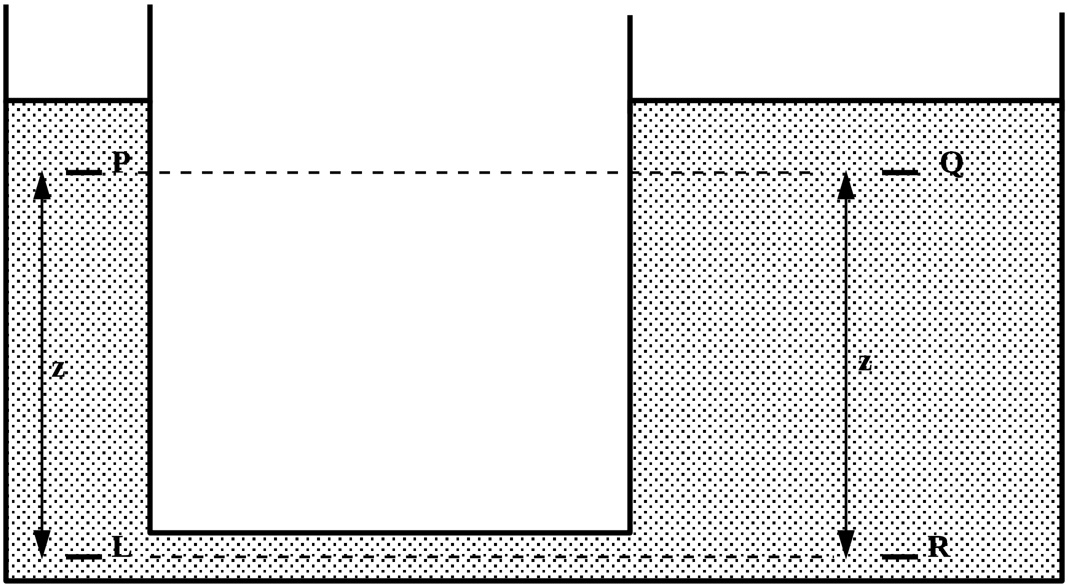
=

=

Pressure in the horizontal direction is _________

This true for any fluid.

Consider these two tanks, one much larger than the other, and linked together by a thin tube:



We have shown

$$p_l = p_r$$

For a vertical pressure change we have

$$p_l =$$

and

$$p_r =$$

so

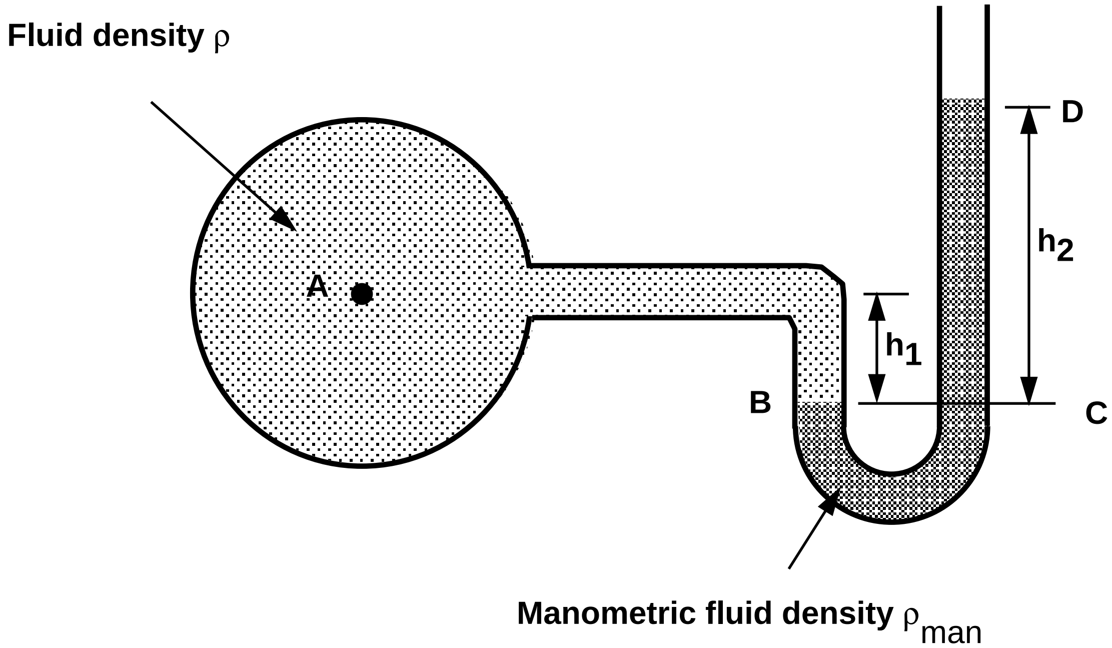
$$=$$

$$p_p = p_q$$

Pressure at the two equal levels are _______.

An improved manometer: The "U"-Tube

"U" is connected as shown and filled with *manometric fluid*.



Using the fact that pressure at two levels are equal:

pressure atB **=**

p_B =

For the left hand arm

pressure at B **=**

p_B =

For the right hand arm

pressure at C **=**

p_C =

$p_B = p_C$

p_A =

Subtract $p_{atmospheric}$ to give *gauge pressure*

p_A =

“U”-Tube enables the pressure of both liquids and gases to be measured

Important points:

1. “U”-Tube enables the pressure of both liquids and gases to be measured

2. The manometric fluid density should be ________ ____ ____ _______ measured.

$$\rho_{man} > \rho$$

3. The two fluids should __________ _____ ___ they must be immiscible.

What if the fluid is a gas?

_________ changes.

The manometric fluid is liquid
(usually mercury, oil or water)

And Liquid density is much greater than gas,

$$\rho_{man} >> \rho$$

$\rho g h_2$ is negligible and pressure is given by

$$p_A =$$

An example of the U-Tube manometer.

Using a u-tube manometer to measure gauge pressure of fluid density ρ = 700 kg/m^3, and the manometric fluid is mercury, with a relative density of 13.6.

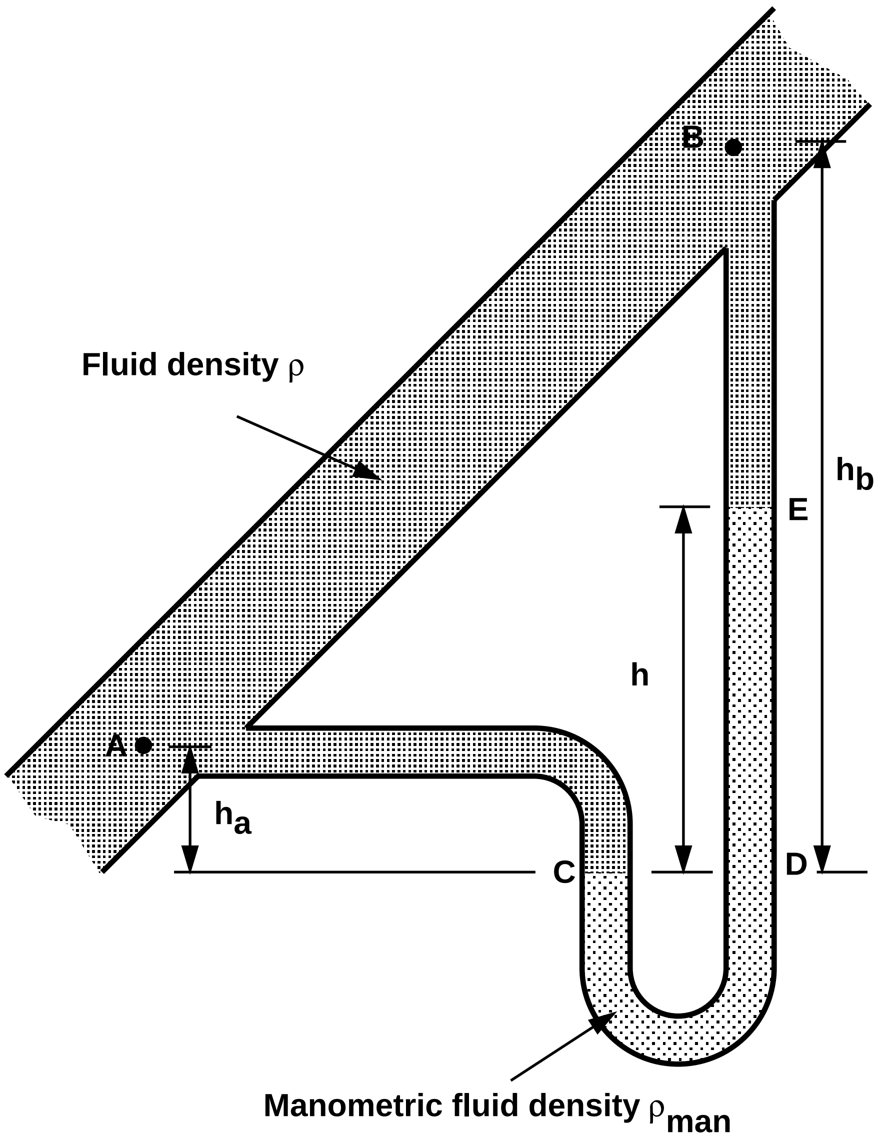
What is the gauge pressure if:

a) h_1 = 0.4m and h_2 = 0.9m?

b) h_1 = 0.4 and h_2 = -0.1m?

Pressure Difference Measurement Using a "U"-Tube Manometer.

The "U"-tube manometer can be connected at both ends to measure *pressure difference* between these two points



pressure at C =

$$p_C =$$

$$p_C =$$

$$p_D =$$

$$=$$

Giving the pressure difference

$$p_A - p_B =$$

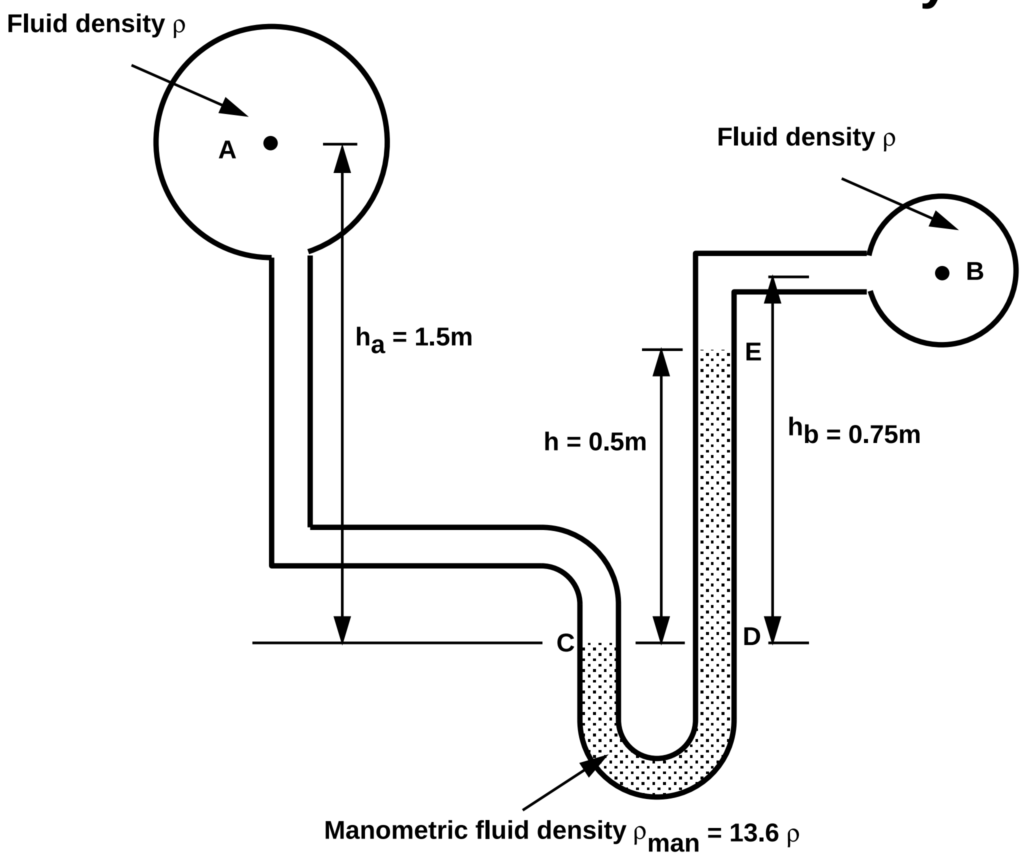
Again if the fluid is a gas $\rho_{man} >> \rho$, then the terms involving ρ can be neglected,

$$p_A - p_B =$$

An example using the u-tube for pressure difference measuring

In the figure below two pipes containing the same fluid of density ρ = 990 kg/m^3 are connected using a u-tube manometer.

a) What is the pressure between the two pipes if the manometer contains fluid of relative density 13.6?

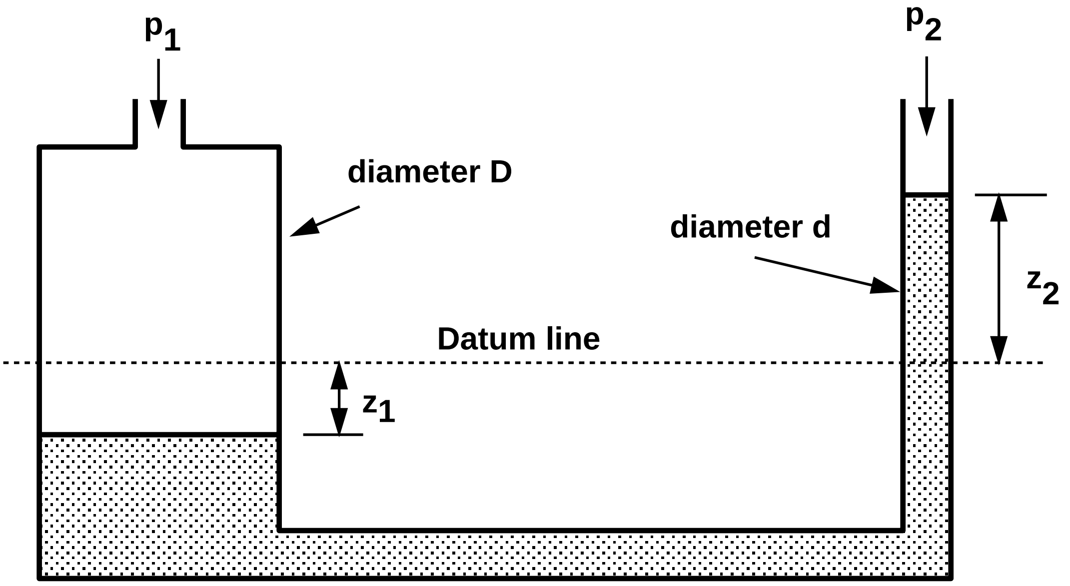


Advances to the "U" tube manometer

Problem: Two reading are required.

Solution:

Result:



If the manometer is measuring the pressure difference of a gas of *(p_1 - p_2)* as shown, we know

$$p_1 - p_2 =$$

volume of liquid moved from the left side to the right

=

The fall in level of the left side is

$$z_1 = \rule{4cm}{0.4pt}$$

$$= \frac{z_2\left(\pi d^2 / 4\right)}{\pi D^2 / 4}$$

$$= z_2\left(\frac{d}{D}\right)^2$$

Putting this in the equation,

$$p_1 - p_2 = \rho g\left[z_2 + z_2\left(\frac{d}{D}\right)^2\right]$$

$$= \rho g z_2\left[1 + \left(\frac{d}{D}\right)^2\right]$$

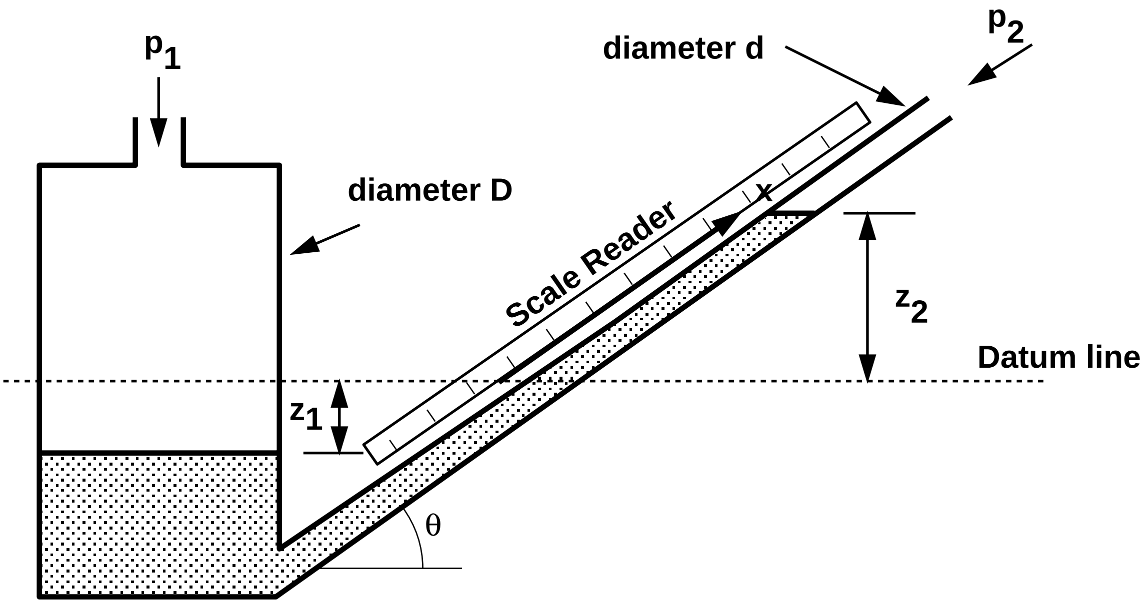
If D >> d then (*d/D*)^2 is very small so

$$p_1 - p_2 =$$

Inclined manometer

Problem: Small changes difficult to see

Incline the arm: same height change but bigger movement.



The pressure difference is still given by the height change of the manometric fluid.

$$p_1 - p_2 = \rho g z_2$$

but,

$$z_2 =$$

$$p_1 - p_2 =$$

The sensitivity to pressure change can be __________ further by a ___________ inclination.

Example of an inclined manometer.

An inclined tube manometer consists of a vertical cylinder 35mm diameter. At the bottom of this is connected a tube 5mm in diameter inclined upward at an angle of 15° to the horizontal, the top of this tube is connected to an air duct. The vertical cylinder is open to the air and the manometric fluid has relative density 0.785.

a) Determine the pressure in the air duct if the manometric fluid moved 50mm along the inclined tube.

a) What is the error if the movement of the fluid in the vertical cylinder is ignored?

Choice Of Manometer

Take care when fixing the manometer to vessel

Burrs cause local pressure variations.

Disadvantages:

- **______ response - only really useful for very slowly varying pressures - no use at all for fluctuating pressures;**
- **For the "U" tube manometer ____ measurements must be taken ________________ to get the h value.**
- **It is often difficult to measure ______ variations in pressure.**
- **It cannot be used for ____ _____ pressures unless several manometers are connected in series;**
- **For very accurate work the ______________ and relationship between temperature and ρ must be known;**

Advantages of manometers:

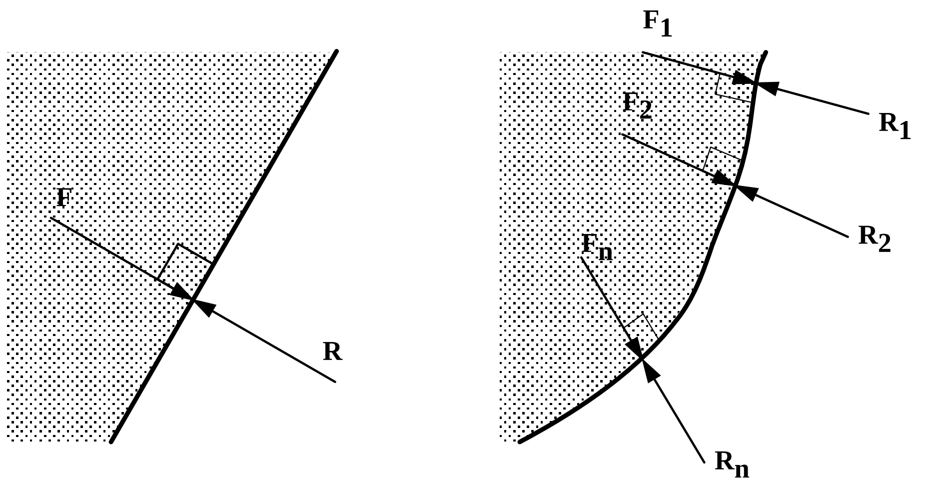
- **They are very __________.**
- **No _______________ is required - the pressure can be calculated from first principles.**

Lecture 5: Forces in Static Fluids
Unit 2: Statics

From earlier we know that:

1. A static fluid can have _______________ acting on it.

2. Any force between the fluid and the boundary must ____.



Pressure force normal to the boundary. True also for

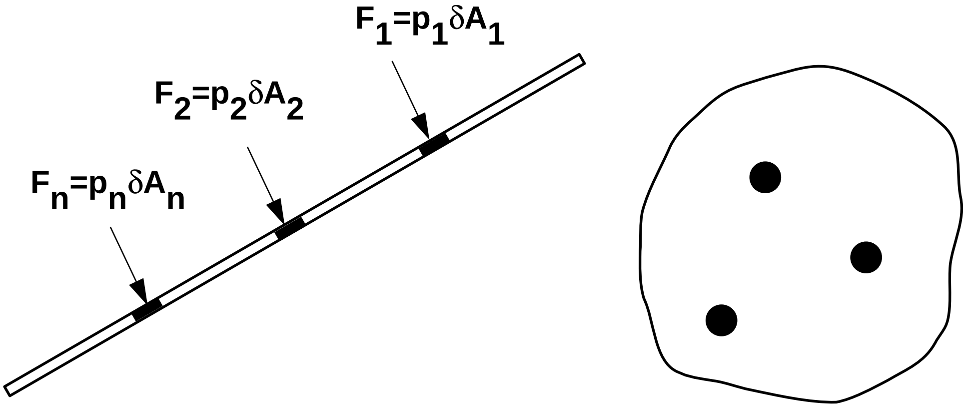
- **curved surfaces**
- **any imaginary plane in the fluid**

An element of fluid at rest is in equilibrium:

3. The sum of forces in any direction is ______

4. The sum of the moments of forces about any point is _______

General submerged plane



The total or *resultant* force, *R*, on the plane is the sum of the forces on the small elements i.e.

$R =$

and

This ____________force will act through the *centre of* __________*.*

For a plane surface all forces acting can be represented by one single ____________ *force,* acting at right-angles to the plane through the *centre of* ____________.

Horizontal submerged plane

The pressure, *p*, will be _______ at all points of the surface.

The resultant force will be given by

$$R =$$

$$R =$$

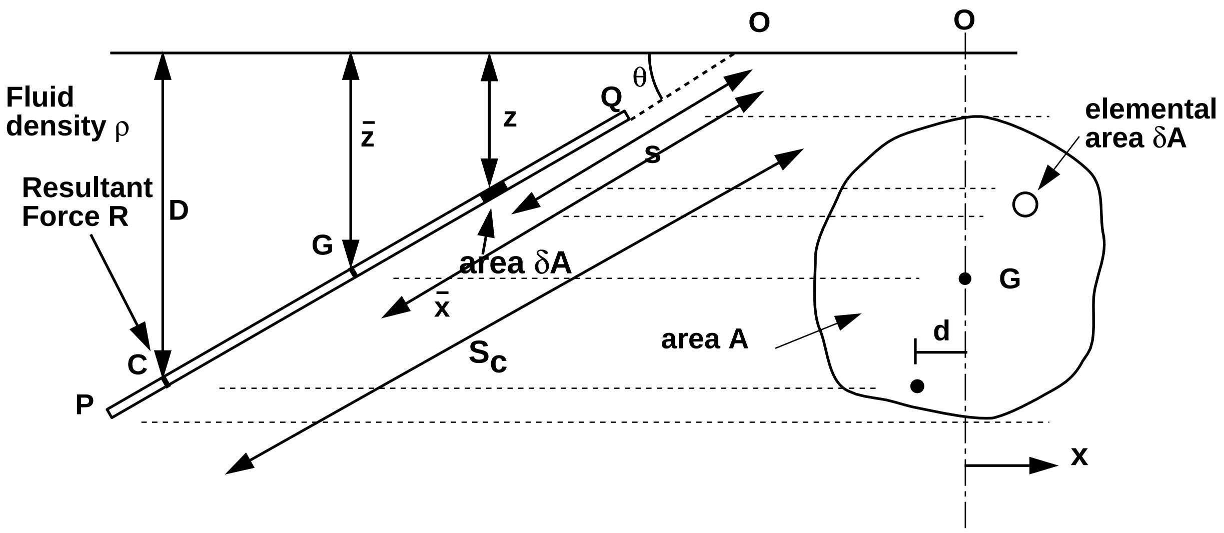
Curved submerged surface

Each elemental force is a different magnitude and in a different direction (but still normal to the surface.).

It is, in general, not easy to calculate the resultant force for a curved surface by combining all elemental forces.

The sum of all the forces on each element will always **be _____ than the sum of the individual forces,** $\sum p\delta A$**.**

Resultant Force and Centre of Pressure on a general plane surface in a liquid.



Take pressure as zero at the surface.

Measuring down from the surface, the pressure on an element δA, depth z,

$$p =$$

So force on element

$$F =$$

Resultant force on plane

$$R =$$

(assuming ρ and g as constant).

$\sum z\delta A$ is known as the 1st *Moment of Area* of the plane PQ about the free surface.

And it is known that $\sum z\delta A =$

A is the area of the plane

$\bar{z}$ is the distance to the *centre of* __________ (____________)

In terms of distance from point O

$$\sum z\delta A =$$

=

(as $\bar{z} = \bar{x}\sin\theta$)

The resultant force on a plane

$$R =$$

$$= -$$

R = Pressure at centre of gravity X Area

This resultant force acts at right angles through the centre of pressure, C, at a depth D.

How do we find this position?

_______________________________.

As the plane is in equilibrium:

The moment of *R* will be equal to ________ ______ ____________________ on all the elements δA about the same point.

It is convenient to take moment about O

The force on each elemental area:

$$\text{Force on } \delta A =$$

$$=$$

the moment of this force is:

$$\text{Moment of Force on } \delta A \text{ about O} = \rho g\, s \sin\theta\, \delta A \times s$$

$$= \rho g \sin\theta\, \delta A s^2$$

ρ , g and θ are the same for each element, giving the total moment as

Sum of moments =

Moment of R about O =

Equating

=

The position of the centre of ________ along the plane measure from the point O is:

$$S_c = \frac{\sum s^2 \delta A}{A\bar{x}}$$

How do we work out the summation term?

This term is known as the "*Second Moment of Area"* , I_o, of the plane (about an axis through O)

$$\text{2nd moment of area about O} = I_o = \sum s^2 \delta A$$

It can be easily calculated for many common shapes.

The position of the *centre of pressure* along the plane measure from the point O is:

$$S_c = \frac{2nd\ \text{Moment of area about a line through O}}{2nd\ \text{Moment of area about a line through O}}$$

and

Depth to the *centre of pressure* is

D =

How do you calculate the 2nd moment of area?

2nd moment of area is a geometric property.

It can be found from tables -
BUT only for moments about
an axis through its centroid = I_{GG}.

We need it for an axis through O

Use the *parallel axis theorem*
to give us what we want.

The *parallel axis theorem* can be written

$$I_o = \qquad\qquad\qquad -$$

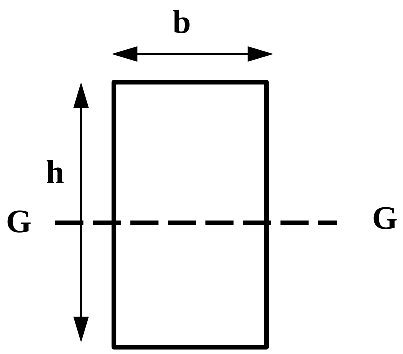
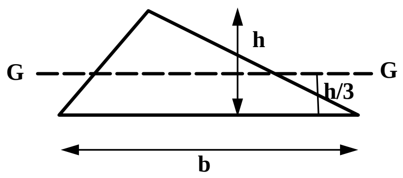
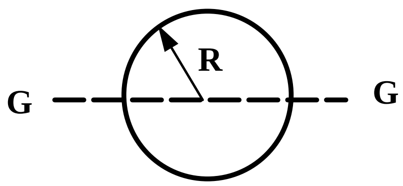
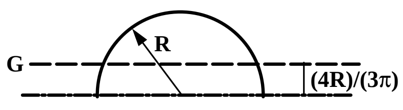
We then get the following equation for the position of the centre of pressure

$$S_c = \frac{\quad}{\quad}$$

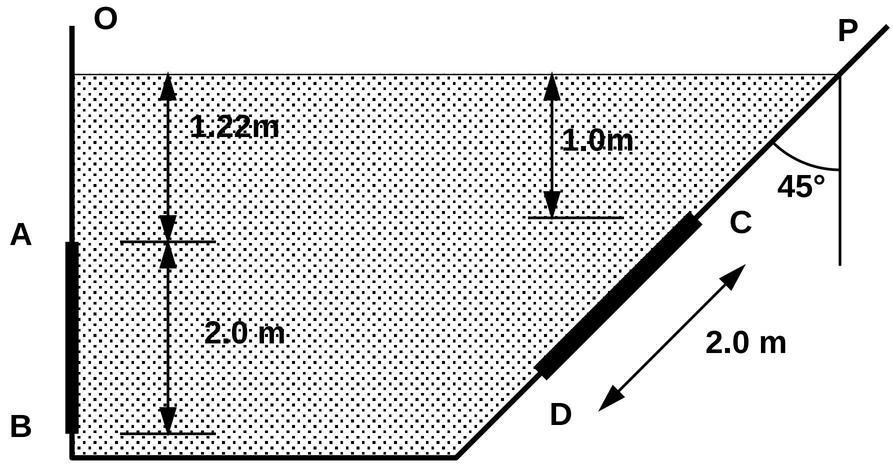
$$D = \quad \left(\frac{\quad}{\quad} \qquad \right)$$

(In the examination the parallel axis theorem and the I_{GG} will be given)

The 2nd moment of area about a line through the centroid of some common shapes.

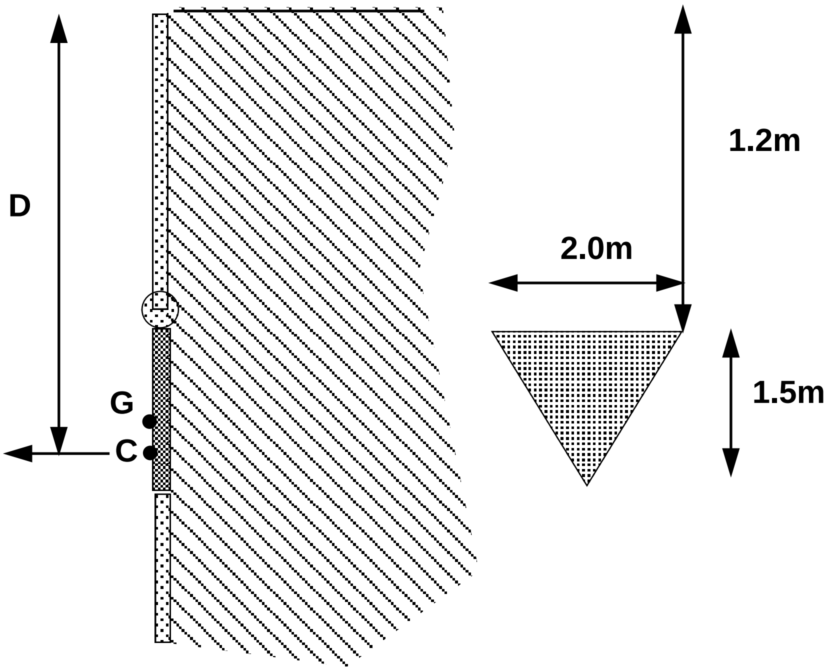
Shape	Area A	2nd moment of area, I_{GG}, about an axis through the centroid
Rectangle 	bd	$\dfrac{bd^3}{12}$
Triangle 	$\dfrac{bd}{2}$	$\dfrac{bd^3}{36}$
Circle 	πR^2	$\dfrac{\pi R^4}{4}$
Semicircle 	$\dfrac{\pi R^2}{2}$	$0.1102R^4$

Example 1: Determine the resultant force due to the water acting on the 1m by 2m rectangular area AB shown in the diagram below. [43 560 N, 2.37m from O



Example 2: Determine the resultant force due to the water acting on the 1.25m by 2.0m triangular area CD shown in the example above. The apex of the triangle is at C. [23.8×10^3N, 2.821m from P]

Example 3: Find the moment required to keep this triangular gate closed on a tank which holds water.



Lecture 6: Pressure Diagrams and Forces on Curved Surfaces
Unit 2: Statics

Pressure diagrams

For vertical walls of constant width it is possible to find the resultant force and centre of pressure graphically using a *pressure diagram*.

We know the relationship between pressure and depth:

$$p =$$

So we can draw the diagram below:

This is know as a *pressure diagram*.

Pressure increases from zero at the surface linearly by *p* = , to a maximum at the base of *p* = .

The *area* of this triangle represents the *resultant force* ___ ____ ____ on the vertical wall,

Units of this are _________ per metre.

Area =

=

=

Resultant force per unit width

$$R = \overline{}$$

The force acts through the *centroid of the pressure diagram*.

For a triangle the centroid is at its height

i.e. the resultant force acts horizontally through the point z = .

For a vertical plane the depth to the centre of pressure is given by

D =

Check this against the moment method:

The resultant force is given by:

R =

=

=

and the depth to the centre of pressure by:

$$D = \qquad \left(\frac{\qquad}{\qquad} \right)$$

and by the parallel axis theorem (with width of 1)

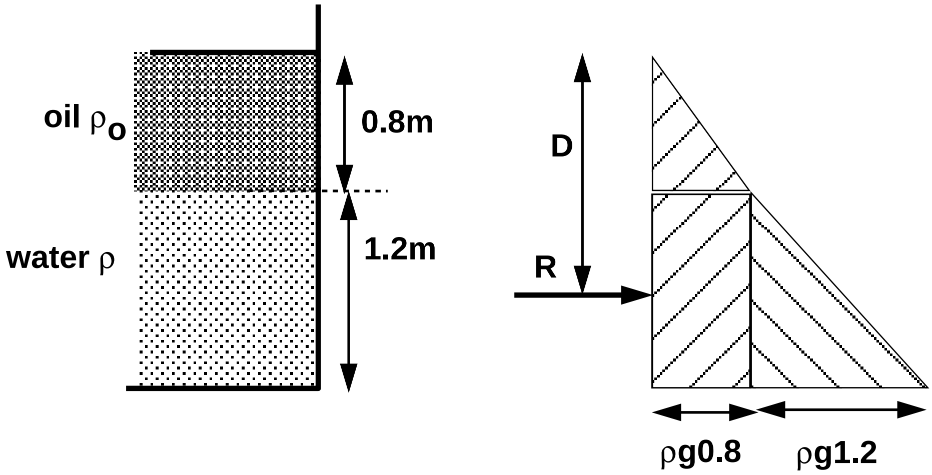
$$I_o = I_{GG} + A\overline{x}^2$$

$$= -$$

Depth to the centre of pressure

$$D = -$$

The same technique can be used with combinations of liquids are held in tanks (e.g. oil floating on water). For example:



Find the position and magnitude of the resultant force on this vertical wall of a tank which has oil floating on water as shown.

Forces on Submerged Curved Surfaces

If the surface is curved the resultant force must be found by combining the elemental forces using some vectorial method.

Calculate the

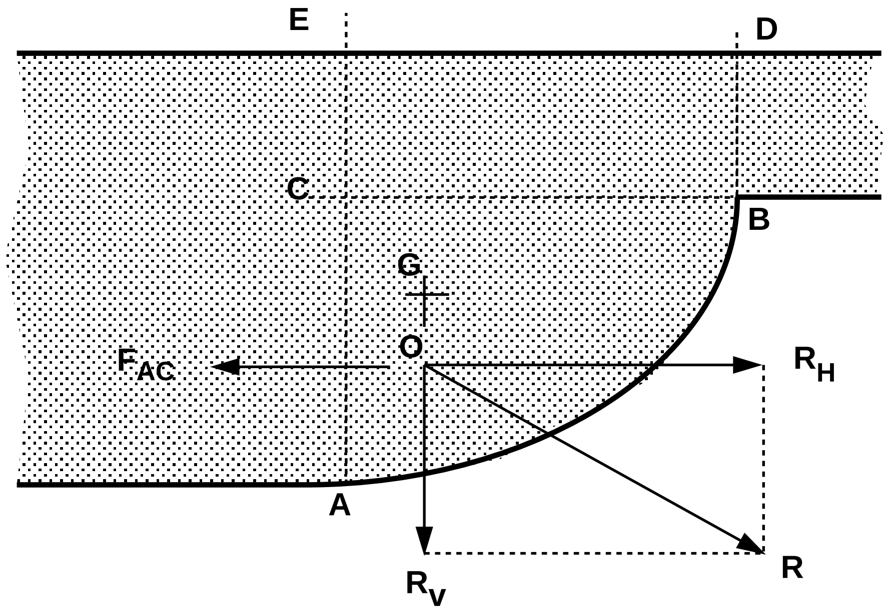
_____________ *and* _____________

components.

Combine these to obtain the resultant force and direction.

(Although this can be done for all three dimensions we will only look at one vertical plane)

In the diagram below liquid is resting on top of a curved base.

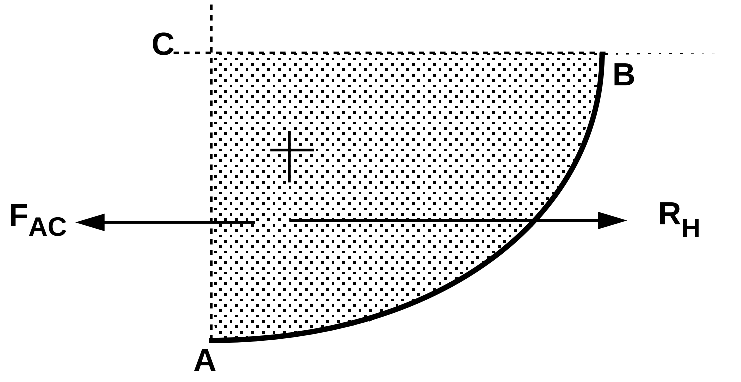


The fluid is at rest – in equilibrium.

So ____ ___________ of fluid such as ABC is also in _____________.

Consider the Horizontal forces

The sum of the horizontal forces is zero.



No horizontal force on CB as there are no shear forces in a static fluid

Horizontal forces act only on the faces AC and AB as shown.

F_{AC}, must be equal and opposite to R_H.

AC is the projection of the curved surface AB onto a vertical plane.

The resultant horizontal force of a fluid above a curved surface is:

R_H =

____

We know

1. The force on a vertical plane must act horizontally (as it acts normal to the plane).

2. That R_H must act through the same point.

So:

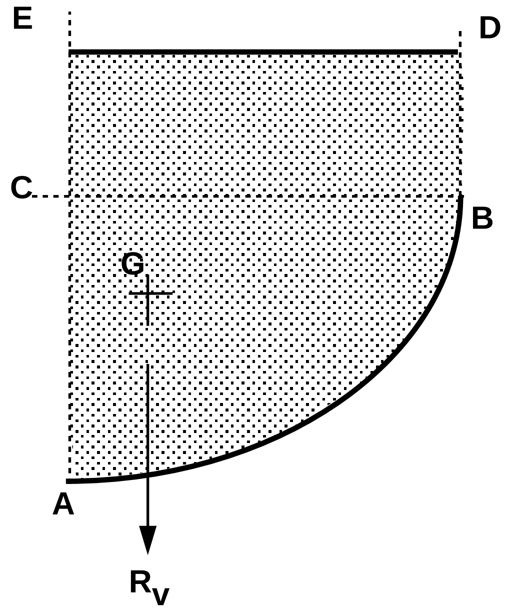
R_H acts horizontally through the ________ _______ _________ of the _______________ of the curved surface onto an vertical plane.

We have seen earlier how to calculate resultant forces and point of action.

Hence we can calculate the resultant horizontal force on a curved surface.

Consider the Vertical forces

The sum of the vertical forces is zero.



There are no shear force on the vertical edges, so the vertical component can only be due to the ___________ *of the fluid*.

So we can say

The resultant vertical force of a fluid above a curved surface is:

***R_V* =**

______________________________.

It will act vertically down through the centre of gravity of the mass of fluid.

Resultant force

The overall resultant force is found by combining the vertical and horizontal components vectorialy,

Resultant force

$$R = \qquad\qquad -$$

And acts through O at an angle of θ.

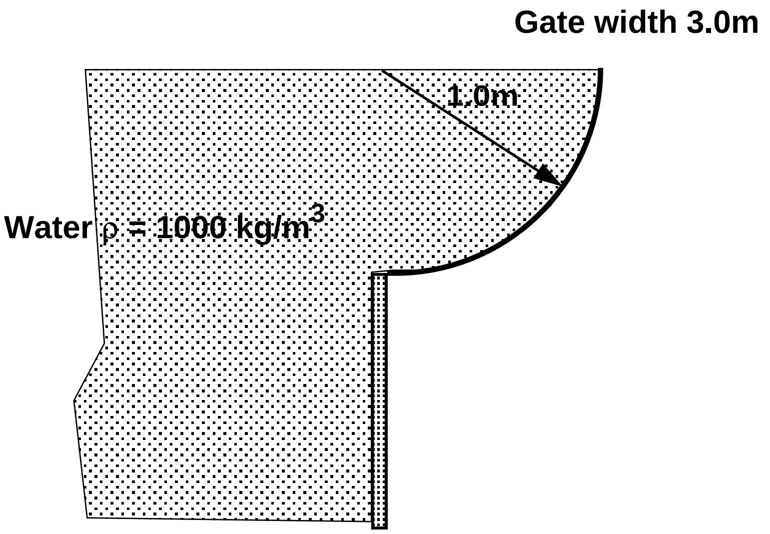
The angle the resultant force makes to the horizontal is

$$\theta =$$

The position of O is the point of intersection of the horizontal line of action of R_H and the vertical line of action of R_V.

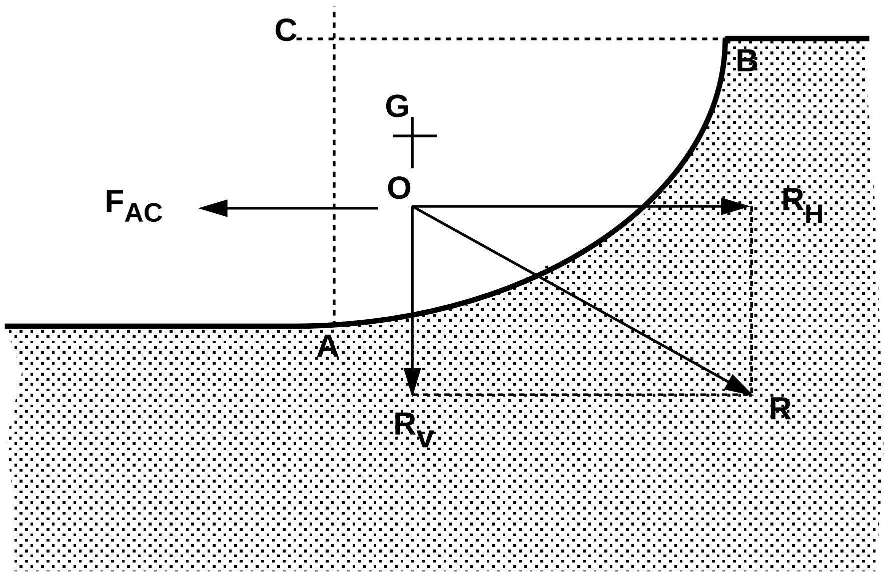
A typical example application of this is the determination of the forces on dam walls or curved sluice gates.

Find the magnitude and direction of the resultant force of water on a quadrant gate as shown below.



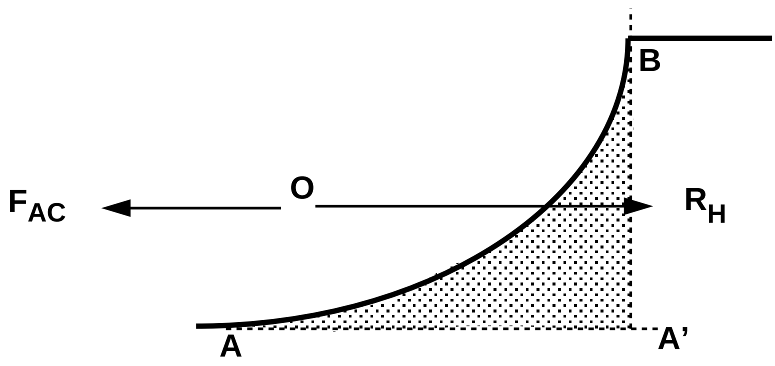
What are the forces if the fluid is *below* ***the curved surface?***

This situation may occur or a curved sluice gate.



The force calculation is very similar to when the fluid is above.

Horizontal force



The two horizontal on the element are:

The horizontal reaction force R_H

The force on the vertical plane A'B.

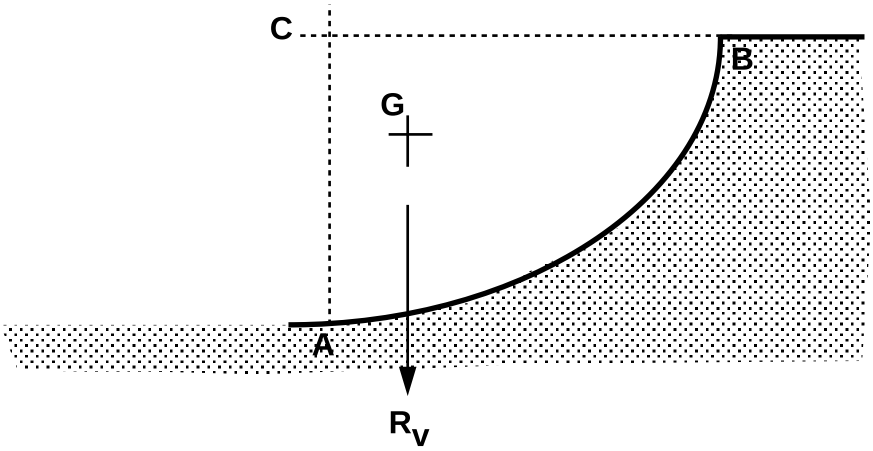
The resultant horizontal force, R_H acts as shown in the diagram. Thus we can say:

The resultant horizontal force of a fluid below a curved surface is:

R_H =

____

Vertical force



What vertical force would keep this in equilibrium?

If the region above the curve were all water there would be equilibrium.

Hence: the force exerted by this amount of fluid must equal he resultant force.

The resultant vertical force of a fluid below a curved surface is:

R_v = Weight of the ______________ volume of fluid ______________ __________the curved surface.

The resultant force and direction of application are calculated in the same way as for fluids above the surface:

Resultant force

$$R =$$

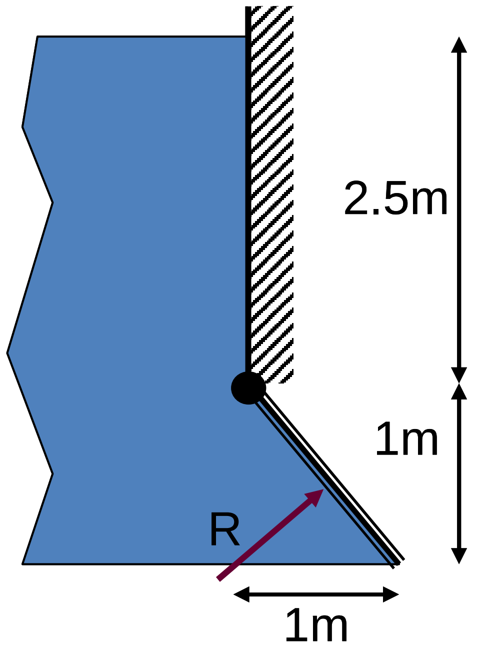
And acts through O at an angle of θ.

The angle the resultant force makes to the horizontal is

$$\theta =$$

Example 1:

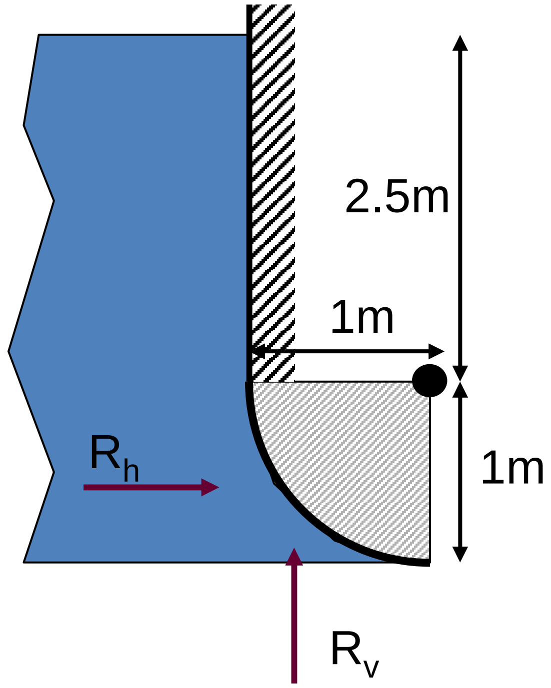
A 2m wide wall as shown in the figure below is fitted with a sluice gate at the base. Find the force on the gate.



Force on a submerged surface,

R = pressure at centroid $\times$ Area of surface

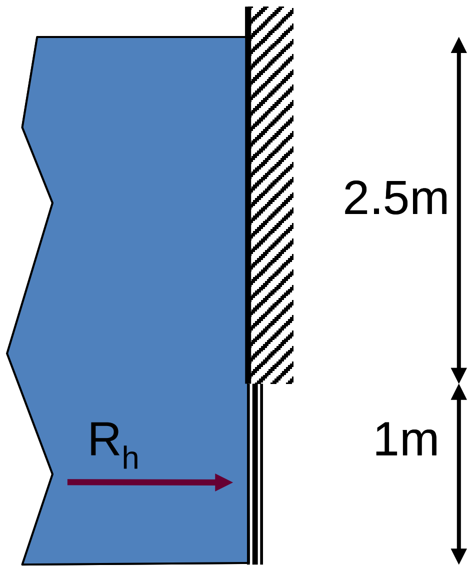
Exampel 2: What would be the force if the gate was changed to a circular one arranged as below?



Horizontal force on curved surface =

force on projection on to a vertical surface

i.e. equivalent to this figure:



$\mathbf{R_h = \rho g h A =}$

$\mathbf{R_h =}$

Vertical force = weight of (real or imaginary) fluid above the surface.

$$R_v = \rho g \times \text{Volume above surface}$$

$$R_v =$$

$$R_v =$$

$$R = (R_h^2 + R_v^2)^{1/2} =$$

Example 3: A curved sluice gate which experiences force from fluid below.

A 1.5m long cylinder lies as shown in the figure, holding back oil of relative density 0.8. If the cylinder has a mass of 2250 kg find

a) the reaction at A   b) the reaction at B