CIVE1400: An Introduction to Fluid Mechanics

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Module Material on the Web:
Use the VLE
Also: www.efm.leeds.ac.uk/CIVE/FluidsLevel1

Unit 1: Fluid Mechanics Basics — 3 lectures
- Flow
- Pressure
- Properties of Fluids
- Fluids vs. Solids
- Viscosity

Unit 2: Statics — 3 lectures
- Hydrostatic pressure
- Manometry/Pressure measurement
- Hydrostatic forces on submerged surfaces

Unit 3: Dynamics — 7 lectures
- The continuity equation.
- The Bernoulli Equation.
- Application of Bernoulli equation.
- The momentum equation.
- Application of momentum equation.

Unit 4: Effect of the boundary on flow — 4 lectures
- Laminar and turbulent flow
- Boundary layer theory
- An Intro to Dimensional analysis
- Similarity

Notes For the First Year Lecture Course:

An Introduction to Fluid Mechanics

School of Civil Engineering, University of Leeds.
CIVE1400 FLUID MECHANICS
Dr Andrew Sleigh
January 2009

Contents of the Course

Objectives:

The course will introduce fluid mechanics and establish its relevance in civil engineering.

Develop the fundamental principles underlying the subject.

Demonstrate how these are used for the design of simple hydraulic components.

Civil Engineering Fluid Mechanics

Why are we studying fluid mechanics on a Civil Engineering course? The provision of adequate water services such as the supply of potable water, drainage, sewerage is essential for the development of industrial society. It is these services which civil engineers provide.

Fluid mechanics is involved in nearly all areas of Civil Engineering either directly or indirectly. Some examples of direct involvement are those where we are concerned with manipulating the fluid:

- Sea and river (flood) defences;
- Water distribution / sewerage (sanitation) networks;
- Hydraulic design of water/sewage treatment works;
- Dams;
- Irrigation;
- Pumps and Turbines;
- Water retaining structures.

And some examples where the primary object is construction - yet analysis of the fluid mechanics is essential:

- Flow of air in buildings;
- Flow of air around buildings;
- Bridge piers in rivers;
- Ground-water flow – much larger scale in time and space.

Notice how nearly all of these involve water. The following course, although introducing general fluid flow ideas and principles, the course will demonstrate many of these principles through examples where the fluid is water.

Module Consists of:

Lectures:
20 Classes presenting the concepts, theory and application.
Worked examples will also be given to demonstrate how the theory is applied. You will be asked to do some calculations - so **bring a calculator.**

Assessment:
1 Exam of 2 hours, worth 80% of the module credits.
This consists of 6 questions of which you choose 4.

2 Multiple choice question (MCQ) papers, worth 10% of the module credits (5% each).

These will be for 30mins and set after the lectures. The timetable for these MCQs and lectures is shown in the table at the end of this section.

1 Marked problem sheet, worth 10% of the module credits.

Laboratories: 2 x 3 hours
These two laboratory sessions examine how well the theoretical analysis of fluid dynamics describes what we observe in practice.
During the laboratory you will take measurements and draw various graphs according to the details on the laboratory sheets. These graphs can be compared with those obtained from theoretical analysis.
You will be expected to draw conclusions as to the validity of the theory based on the results you have obtained and the experimental procedure.
After you have completed the two laboratories you should have obtained a greater understanding as to how the theory relates to practice, what parameters are important in analysis of fluid and where theoretical predictions and experimental measurements may differ.
The two laboratories sessions are:

1. Impact of jets on various shaped surfaces - a jet of water is fired at a target and is deflected in various directions. This is an example of the application of the momentum equation.
2. The rectangular weir - the weir is used as a flow measuring device. Its accuracy is investigated. This is an example of how the Bernoulli (energy) equation is applied to analyses fluid flow.

[As you know, these laboratory sessions are **compulsory** course-work. You must attend them. Should you fail to attend either one you will be asked to complete some extra work. This will involve a detailed report and further questions. The simplest strategy is to do the lab.]

Homework:
Example sheets: These will be given for each section of the course. Doing these will **greatly** improve your exam mark. They are course work but do not have credits toward the module.
Lecture notes: Theses should be studied but explain only the basic outline of the necessary concepts and ideas.
Books: It is very important do some extra reading in this subject. To do the examples you will definitely need a textbook. Any one of those identified below is adequate and will also be useful for the fluids (and other) modules in higher years - and in work.

Example classes:
There will be example classes each week. You may bring any problems/questions you have about the course and example sheets to these classes.

Schedule:

Lecture	Month	Date	Week	Day	Time	Unit	
1	January	20	0	Tues	3.00 pm	**Unit 1:** Fluid Mechanic Basics	Pressure, density
2		21	0	Wed	9.00 am		Viscosity, Flow
Extra		27	1	Tues	3.00 pm	Presentation of Case Studies	double lecture
3		28	1	Wed	9.00 am		Flow calculations
4		3	2	Tues	3.00 pm	**Unit 2:** Fluid Statics	Pressure
5		4	2	Wed	9.00 am		Plane surfaces
6	February	10	3	Tues	3.00 pm		Curved surfaces
7		11	3	Wed	9.00 am	Design study 01 - Centre vale park	
8		17	4	Tues	3.00 pm	**Unit 3:** Fluid Dynamics	General Descrip.
MCQ					4.00 pm	MCQ	
9		18	4	Wed	9.00 am		Bernoulli
10		24	5	Tues	3.00 pm		Flow measurment
11		25	5	Wed	9.00 am		Weir
12	March	3	6	Tues	3.00 pm		Momentum
13	surveying	4	6	Wed	9.00 am	Design study 02 - Gaunless + Millwood	
12		10	7	Tues	3.00 pm		Momentum
13		11	7	Wed	9.00 am	Design study 02 - Gaunless + Millwood	
14		17	8	Tues	3.00 pm		Applications
15		18	8	Wed	9.00 am	problem sheet given out	Calculation
			Vacation				
16	April	21	9	Tues	3.00 pm	**Unit 4:** Effects of the Boundary on Flow	Boundary Layer
17		22	9	Wed	9.00 am		Laminar flow
18		28	10	Tues	3.00 pm		Dim. Analysis
19		29	10	Wed	9.00 am	problem sheet handed in	Dim. Analysis
20	May	5	11	Tues	3.00 pm		Boundary layers
MCQ					4.00 pm	MCQ	
21		6	11	Wed	9.00 am	Revision	

Books:

Any of the books listed below are more than adequate for this module. (You will probably not need any more fluid mechanics books on the rest of the Civil Engineering course)

Mechanics of Fluids, Massey B S., Van Nostrand Reinhold.

Fluid Mechanics, Douglas J F, Gasiorek J M, and Swaffield J A, Longman.

Civil Engineering Hydraulics, Featherstone R E and Nalluri C, Blackwell Science.

Hydraulics in Civil and Environmental Engineering, Chadwick A, and Morfett J., E & FN Spon - Chapman & Hall.

Online Lecture Notes:

http://www.efm.leeds.ac.uk/cive/FluidsLevel1

There is a **lot** of extra teaching material on this site: Example sheets, Solutions, Exams, Detailed lecture notes, Online video lectures, MCQ tests, Images etc. This site DOES NOT REPLACE LECTURES or BOOKS.

Take care with the System of Units

As any quantity can be expressed in whatever way you like it is sometimes easy to become confused as to what exactly or how much is being referred to. This is particularly true in the field of fluid mechanics. Over the years many different ways have been used to express the various quantities involved. Even today different countries use different terminology as well as different units for the same thing - they even use the same name for different things e.g. an American pint is 4/5 of a British pint!

To avoid any confusion on this course we will always use the SI (metric) system - which you will already be familiar with. It is essential that all quantities are expressed in the same system or the wrong solutions will results.

Despite this warning you will still find that this is the most common mistake when you attempt example questions.

The SI System of units

The SI system consists of six **primary** units, from which all quantities may be described. For convenience **secondary** units are used in general practice which are made from combinations of these primary units.

Primary Units

The six **primary** units of the SI system are shown in the table below:

Quantity	SI Unit	Dimension
Length	metre, m	L
Mass	kilogram, kg	M
Time	second, s	T
Temperature	Kelvin, K	θ
Current	ampere, *A*	I
Luminosity	candela	Cd

In fluid mechanics we are generally only interested in the top four units from this table.

Notice how the term 'Dimension' of a unit has been introduced in this table. This is not a property of the individual units, rather it tells what the unit represents. For example a metre is a length which has a dimension L but also, an inch, a mile or a kilometre are all lengths so have dimension of L.

(The above notation uses the MLT system of dimensions, there are other ways of writing dimensions - we will see more about this in the section of the course on dimensional analysis.)

Derived Units

There are many **derived** units all obtained from combination of the above **primary** units. Those most used are shown in the table below:

Quantity	SI Unit		Dimension
Velocity	m/s	ms^{-1}	LT^{-1}
acceleration	m/s^2	ms^{-2}	LT^{-2}
force	N $kg\ m/s^2$	$kg\ ms^{-2}$	$M\ LT^{-2}$
energy (or work)	Joule J N m, $kg\ m^2/s^2$	$kg\ m^2s^{-2}$	ML^2T^{-2}
power	Watt W N m/s $kg\ m^2/s^3$	Nms^{-1} $kg\ m^2s^{-3}$	ML^2T^{-3}
pressure (or stress)	Pascal P, N/m^2, $kg/m/s^2$	Nm^{-2} $kg\ m^{-1}s^{-2}$	$ML^{-1}T^{-2}$
density	kg/m^3	$kg\ m^{-3}$	ML^{-3}
specific weight	N/m^3 $kg/m^2/s^2$	$kg\ m^{-2}s^{-2}$	$ML^{-2}T^{-2}$
relative density	a ratio no units		1 no dimension
viscosity	$N\ s/m^2$ kg/m s	$N\ sm^{-2}$ $kg\ m^{-1}s^{-1}$	$M\ L^{-1}T^{-1}$
surface tension	N/m $kg\ /s^2$	Nm^{-1} $kg\ s^{-2}$	MT^{-2}

The above units should be used at all times. Values in other units should NOT be used without first converting them into the appropriate SI unit. If you do not know what a particular unit means - **find out**, else your guess will probably be wrong.
More on this subject will be seen later in the section on dimensional analysis and similarity.

Properties of Fluids: Density

There are three ways of expressing density:

1. Mass density:

$$\rho = \text{ mass per unit volume}$$

$$\rho = \frac{\text{mass of fluid}}{\text{volume of fluid}}$$

(units: kg/m3)

2. Specific Weight:

(also known as specific gravity)

$$\omega = \text{ weight per unit volume}$$

$$\omega = \rho g$$

(units: N/m^3 or $kg/m^2/s^2$)

3. Relative Density:

$$\sigma = \text{ ratio of mass density to a standard mass density}$$

$$\sigma = \frac{\rho_{substance}}{\rho_{H_2O(at 4^\circ c)}}$$

For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4°c) at atmospheric pressure.

(units: none, as it is a ratio)

Pressure

Convenient to work in terms of pressure, p, which is the force per unit area.

$$\text{pressure} = \frac{\text{Force}}{\text{Area over which the force is applied}}$$

$$p = \frac{F}{A}$$

Units: Newtons per square metre,
N/m^{2}**,** ***kg/m s***^{2} **(*kg m***$^{-1}$***s***$^{-2}$**).**

Also known as a Pascal, *Pa*, i.e. 1 *Pa* = 1 *N/m*^{2}

Also frequently used is the alternative SI unit the *bar*, where 1*bar* = 10^{5} ***N/m***^{2}

Standard atmosphere = 101325 Pa = 101.325 kPa

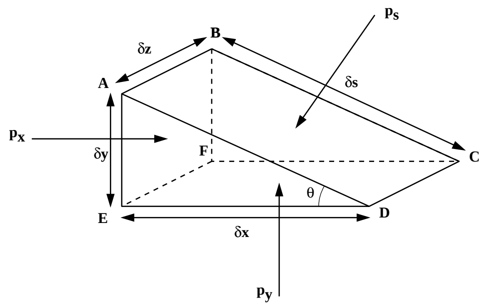
1 bar = 100 kPa (kilopascals)

1 mbar = 0.001 bar = 0.1 kPa = 100 Pa

Uniform Pressure:

If the pressure is the same at all points on a surface *uniform* pressure

Pascal's Law: pressure acts equally in all directions.



No shearing forces :
All forces at right angles to the surfaces

Summing forces in the x-direction:

Force in the x-direction due to p_x,

$$F_{x_x} = p_x \times Area_{ABFE} = p_x \delta x\, \delta y$$

Force in the x-direction due to p_s,

$$F_{x_s} = -p_s \times Area_{ABCD} \times \sin\theta$$
$$= -p_s \delta s\, \delta z \frac{\delta y}{\delta s}$$
$$= -p_s \delta y\, \delta z$$

($\sin\theta = {}^{\delta y}\!/\!_{\delta s}$)

Force in x-direction due to p_y,

$$F_{x_y} = 0$$

To be at rest (in equilibrium) sum of forces is zero

$$F_{x_x} + F_{x_s} + F_{x_y} = 0$$
$$p_x \delta x \delta y + \left(-p_s \delta y \delta z\right) = 0$$
$$p_x = p_s$$

Summing forces in the y-direction.

Force due to p_y,

$$F_{y_y} = p_y \times Area_{EFCD} = p_y \delta x \delta z$$

Component of force due to p_s,

$$F_{y_s} = -p_s \times Area_{ABCD} \times \cos\theta$$
$$= -p_s \delta s \delta z \frac{\delta x}{\delta s}$$
$$= -p_s \delta x \delta z$$

($\cos\theta = {}^{\delta x}\!/\!_{\delta s}$)

Component of force due to p_x,

$$F_{y_x} = 0$$

Force due to gravity,

$$\text{weight} = -\text{specific weight} \times \text{volume of element}$$
$$= -\rho g \times \frac{1}{2} \delta x \delta y \delta z$$

To be at rest (in equilibrium)

$$F_{y_y} + F_{y_s} + F_{y_x} + \text{ weight} = 0$$
$$p_y \delta x \delta y + \left(-p_s \delta x \delta z\right) + \left(-\rho g \frac{1}{2} \delta x \delta y \delta z\right) = 0$$

The element is small i.e. *δx*, *δx*, and *δz*, are small, so *δx × δy × δz*, is <u>very</u> small and considered negligible, hence

$$p_y = p_s$$

We showed above

$$p_x = p_s$$

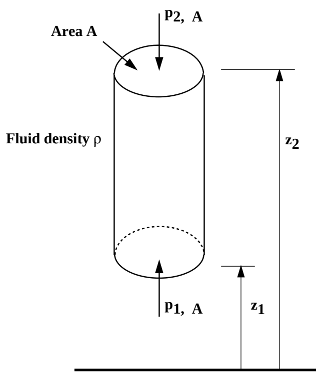
thus

$$p_x = p_y = p_s$$

Pressure at any point is the same in all directions.

This is Pascal's Law **and applies to fluids at rest.**

Change of Pressure in the Vertical Direction



Cylindrical element of fluid, area = *A,* density = ρ

The forces involved are:

Force due to p_1 on A (upward) = p_1A

Force due to p_2 on A (downward) = p_2A

Force due to weight of element (downward)

= *mg= density $\times$ volume $\times$ g*

= $\rho g A(z_2 - z_1)$

Taking upward as positive, we have

$$p_1 A - p_2 A - \rho g A(z_2 - z_1) = 0$$

$$p_2 - p_1 = -\rho g(z_2 - z_1)$$

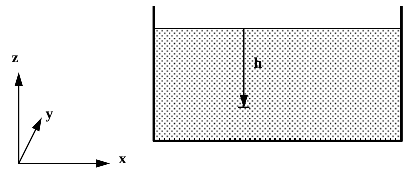
In a fluid pressure decreases linearly with increase in height

$$p_2 - p_1 = -\rho g(z_2 - z_1)$$

This is the hydrostatic pressure change.

With liquids we normally measure *from* the surface.

Measuring *h down* from the free surface so that *h = -z*



giving $p_2 - p_1 = \rho g h$

Surface pressure is atmospheric, $p_{\text{atmospheric}}$.

$$p = \rho g h + p_{\text{atmospheric}}$$

It is convenient to take atmospheric pressure as the datum

Pressure quoted in this way is known as gauge pressure i.e.

Gauge pressure is

$$p_{gauge} = \rho g h$$

The lower limit of any pressure is the pressure in a perfect vacuum.

Pressure measured above a perfect vacuum (zero) is known as absolute pressure

Absolute pressure is

$$p_{absolute} = \rho g h + p_{atmospheric}$$

Absolute pressure = Gauge pressure + Atmospheric

Pressure density relationship

Boyle's Law

$$pV = \text{constant}$$

Ideal gas law

$$pV = nRT$$

where

p **is the absolute pressure, N/m^2, Pa**

V **is the volume of the vessel, m^3**

n **is the amount of substance of gas, moles**

R **is the ideal gas constant,**

T **is the absolute temperature. K**

In SI units, R = 8.314472 J mol^{-1} K^{-1}
(or equivalently m^3 Pa K^{-1} mol^{-1}).

Lecture 2: Fluids vs Solids, Flow

What makes fluid mechanics different to solid mechanics?

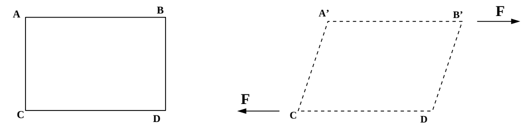
Fluids are clearly different to solids. But we must be specific.

Need definable basic physical difference.

Fluids *flow* under the action of a force, and the solids don't - but solids do deform.

- **fluids lack the ability of solids to resist deformation.**
- **fluids change shape as long as a force acts.**

Take a rectangular element



Forces acting along edges (faces), such as F, are know as *shearing forces*.

A Fluid is a substance which deforms continuously, or flows, when subjected to *shearing forces*.

This has the following implications for <u>fluids at rest</u>:

If a fluid is <u>at rest</u> there are NO shearing forces acting on it, and any force must be acting perpendicular to the fluid

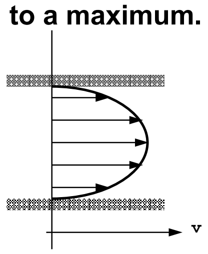
Fluids in motion

Consider a fluid flowing near a wall.
- in a pipe for example -

Fluid next to the wall will have *zero* velocity.

The fluid "*sticks*" to the wall.

Moving away from the wall velocity increases to a maximum.



Plotting the velocity across the section gives "velocity profile"

Change in velocity with distance is

"velocity gradient" = $\frac{du}{dy}$

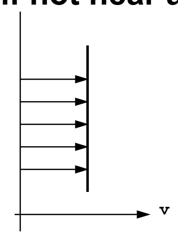
As fluids are usually near surfaces there is usually a velocity gradient.

Under normal conditions one fluid particle has a velocity different to its neighbour.

Particles next to each other with different velocities exert forces on each other (due to intermolecular action) ……

i.e. shear forces exist in a fluid moving close to a wall.

What if not near a wall?



No velocity gradient, no shear forces.

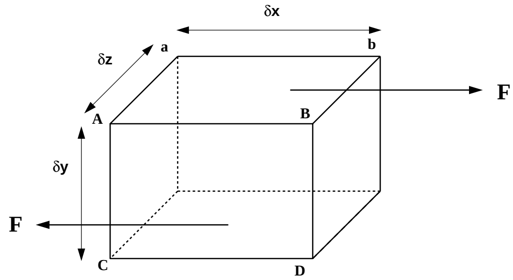
What use is this observation?

It would be useful if we could quantify this shearing force.

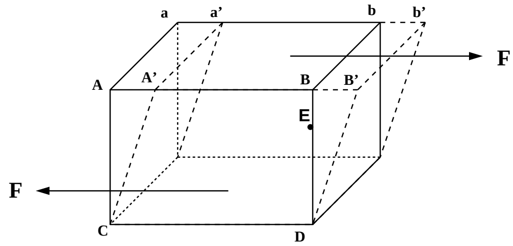
This may give us an understanding of what parameters govern the forces different fluid exert on flow.

We will examine the force required to deform an element.

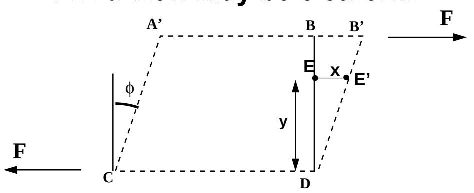
Consider this 3-d rectangular element, under the action of the force F.



under the action of the force F



A 2-d view may be clearer…



The shearing force acts on the area

$$A = \delta z \times \delta x$$

Shear stress, **τ, is the force per unit area:**

$$\tau = \frac{F}{A}$$

The deformation which *<u>shear stress</u>* causes is measured by the angle ϕ, and is know as *<u>shear strain</u>*.

Using these definitions we can amend our definition of a fluid:

In a fluid ϕ increases for as long as τ is applied - the fluid flows **In a solid shear strain, ϕ, is constant for a fixed shear stress τ.**

It has been shown experimentally that the *rate of shear strain* is directly proportional to *shear stress*

$$\tau \propto \frac{\phi}{\text{time}}$$

$$\tau = \text{Constant} \times \frac{\phi}{t}$$

We can express this in terms of the cuboid.

If a particle at point E moves to point E' in time t then:

for small deformations

$$\text{shear strain } \phi = \frac{x}{y}$$

$$\text{rate of shear strain } = -$$

$$= \text{——} = --$$

$$= -$$

(note that $\frac{x}{t} = u$ is the velocity of the particle at E)

So

$$\tau = \text{Constant } \times \frac{u}{y}$$

u/y is the rate of change of velocity with distance,

in differential form this is $\frac{du}{dy}$ = velocity gradient.

The constant of proportionality is known as the *dynamic viscosity,* μ.

giving

$$\tau = \mu \frac{du}{dy}$$

which is know as *Newton's law of viscosity*

A fluid which obeys this rule is know as a *Newtonian Fluid*

(sometimes also called *real fluids)*

Newtonian fluids have constant values of μ

Non-Newtonian Fluids

Some fluids do not have constant μ. They do not obey Newton's Law of viscosity.

They do obey a similar relationship and can be placed into several clear categories

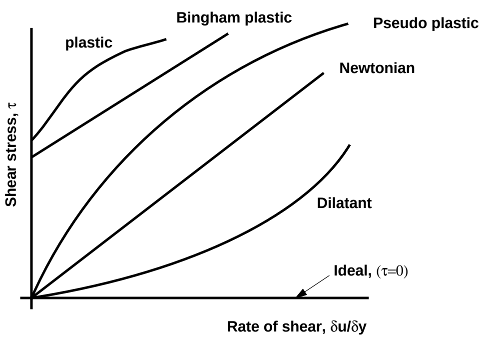
The general relationship is:

$$\tau = A + B\left(\frac{\delta u}{\delta y}\right)^n$$

where A, B and n are constants.

For Newtonian fluids A = 0, B = μ and n = 1

This graph shows how μ changes for different fluids.



- ***Plastic:*** **Shear stress must reach a certain minimum before flow commences.**
- ***Bingham plastic:*** **As with the plastic above a minimum shear stress must be achieved. With this classification n = 1. An example is sewage sludge.**
- ***Pseudo-plastic:*** **No minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. colloidal substances like clay, milk and cement.**
- ***Dilatant substances;*** **Viscosity increases with rate of shear e.g. quicksand.**
- ***Thixotropic substances:*** **Viscosity decreases with length of time shear force is applied e.g. thixotropic jelly paints.**
- ***Rheopectic substances:*** **Viscosity increases with length of time shear force is applied**
- ***Viscoelastic materials:*** **Similar to Newtonian but if there is a sudden large change in shear they behave like plastic**

Viscosity

There are two ways of expressing viscosity

Coefficient of Dynamic Viscosity

$$\mu = \frac{\tau}{du/dy}$$

Units: N s/m^2 or Pa s or kg/m s
The unit Poise is also used where 10 P = 1 Pa·s

Water μ = 8.94 × 10^{-4} Pa s
Mercury μ = 1.526 × 10^{-3} Pa s
Olive oil μ = .081 Pa s
Pitch μ = 2.3 × 10^{8} Pa s
Honey μ = 2000 – 10000 Pa s
Ketchup μ = 50000 – 100000 Pa s (non-newtonian)

Kinematic Viscosity

ν = the ratio of dynamic viscosity to mass density

$$\nu = \frac{\mu}{\rho}$$

Units m^2/s

Water ν = 1.7 × 10^{-6} m^2/s.

Air ν **= 1.5** × 10^{-5} m^2/s.

Flow rate

Mass flow rate

$$\dot{m} = \frac{dm}{dt} = \frac{\text{mass}}{\text{time taken to accumulate this mass}}$$

A simple example:

An empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg, then:

$$\text{mass flow rate} = \dot{m} = \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}}$$
$$= \frac{8.0 - 2.0}{7}$$
$$= 0.857 kg / s$$

Volume flow rate - Discharge.

More commonly we use volume flow rate
Also know as *discharge.*

The symbol normally used for discharge is *Q*.

$$\text{discharge, Q} = \frac{\text{volume of fluid}}{\text{time}}$$

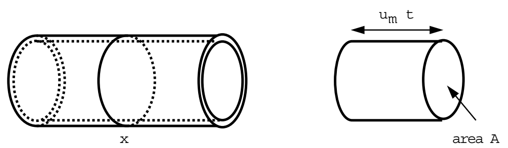
A simple example:

If the bucket above fills with 2.0 litres in 25 seconds, what is the discharge?

$$Q = \frac{2.0 \times 10^{-3}\ m^3}{25 \text{ sec}}$$
$$= 0.0008 m^3 / s$$
$$= 0.8\ l / s$$

Discharge and mean velocity

If we know the discharge and the diameter of a pipe, we can deduce the *mean* velocity



Pipe Cylinder of fluid

Cross sectional area of pipe is *A*
Mean velocity is u_m.

In time *t*, a cylinder of fluid will pass point X with a volume $A \times u_m \times t$.

The discharge will thus be

$$Q = \frac{\text{volume}}{\text{time}} = \frac{A \times u_m \times t}{t}$$
$$Q = A u_m$$

A simple example:

If $A = 1.2 \times 10^{-3} m^2$

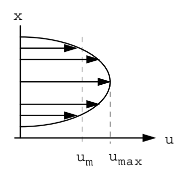
And discharge, *Q* is 24 l/s,

mean velocity is

$$u_m = \frac{Q}{A}$$
$$= \frac{2.4 \times 10^{-3}}{1.2 \times 10^{-3}}$$
$$= 2.0 m / s$$

Note how we have called this the *mean* velocity.

This is because the velocity in the pipe is not constant across the cross section.

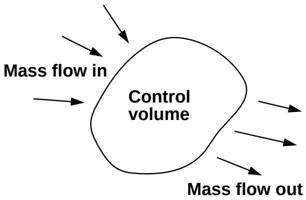


This idea, that mean velocity multiplied by the area gives the discharge, applies to all situations - not just pipe flow.

Continuity

This principle of *conservation of mass* says matter cannot be created or destroyed

This is applied in fluids to fixed volumes, known as control volumes (or surfaces)



For any control volume the principle of *conservation of mass* says

Mass entering per unit time	= Mass leaving per unit time	+	Increase of mass in control vol per unit time

For <u>steady</u> flow there is no increase in the mass within the control volume, so

For steady flow

Mass entering = Mass leaving

In a real pipe (or any other vessel) we use the *mean* velocity and write

$$\rho_1 A_1 u_{m1} = \rho_2 A_2 u_{m2} = \text{ Constant} = \dot{m}$$

For incompressible, fluid $\rho_1 = \rho_2 = \rho$

(dropping the *m* subscript)

$$A_1 u_1 = A_2 u_2 = Q$$

This is the continuity equation most often used.

This equation is a **very powerful** tool.

It will be **used repeatedly** throughout the rest of this course.

Lecture 3: Examples from Unit 1: Fluid Mechanics Basics

Units

1.

A water company wants to check that it will have sufficient water if there is a prolonged drought in the area. The region it covers is 500 square miles and various different offices have sent in the following consumption figures. There is sufficient information to calculate the amount of water available, but unfortunately it is in several different units.

Of the total area 100 000 acres are rural land and the rest urban. The density of the urban population is 50 per square kilometre. The average toilet cistern is sized 200mm by 15in by 0.3m and on average each person uses this 3 time per day. The density of the rural population is 5 per square mile. Baths are taken twice a week by each person with the average volume of water in the bath being 6 gallons. Local industry uses 1000 m^3 per week. Other uses are estimated as 5 gallons per person per day. A US air base in the region has given water use figures of 50 US gallons per person per day.

The average rain fall in 1in per month (28 days). In the urban area all of this goes to the river while in the rural area 10% goes to the river 85% is lost (to the aquifer) and the rest goes to the one reservoir which supplies the region. This reservoir has an average surface area of 500 acres and is at a depth of 10 fathoms. 10% of this volume can be used in a month.

a) What is the total consumption of water per day?
b) If the reservoir was empty and no water could be taken from the river, would there be enough water if available if rain fall was only 10% of average?

Fluid Properties

1. The following is a table of measurement for a fluid at constant temperature. Determine the dynamic viscosity of the fluid.

du/dy (s^{-1})	*0.0*	*0.2*	*0.4*	*0.6*	*0.8*
τ (N m^{-2})	0.0	1.0	1.9	3.1	4.0

2. The density of an oil is 850 kg/m^3. Find its relative density and Kinematic viscosity if the dynamic viscosity is 5×10^{-3} kg/ms.

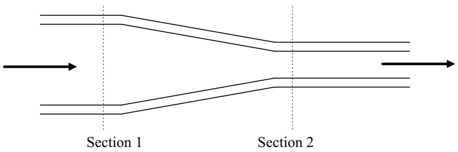
3. The velocity distribution of a viscous liquid (dynamic viscosity μ = 0.9 Ns/m^2) flowing over a fixed plate is given by $u = 0.68y - y^2$ (u is velocity in m/s and y is the distance from the plate in m).
What are the shear stresses at the plate surface and at y=0.34m?

4. 5.6m^3 of oil weighs 46 800 N. Find its mass density, ρ and relative density, γ.

5. From table of fluid properties the viscosity of water is given as 0.01008 poises. What is this value in Ns/m^2 and Pa s units?

6. In a fluid the velocity measured at a distance of 75mm from the boundary is 1.125m/s. The fluid has absolute viscosity 0.048 Pa s and relative density 0.913. What is the velocity gradient and shear stress at the boundary assuming a linear velocity distribution.

Continuity



A liquid is flowing from left to right.

By continuity

$$A_1 u_1 \rho_1 = A_2 u_2 \rho_2$$

As we are considering a liquid (incompressible),

$$\rho_1 = \rho_2 = \rho$$

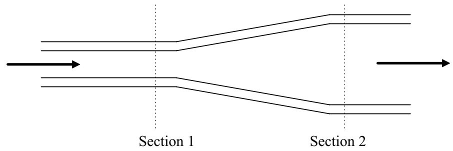
$$Q_1 = Q_2$$

$$A_1 u_1 = A_2 u_2$$

If the area $A_1 = 10\times10^{-3}\ m^2$ and $A_2 = 3\times10^{-3}\ m^2$
And the upstream mean velocity $u_1 = 2.1\ m/s$.

What is the downstream mean velocity?

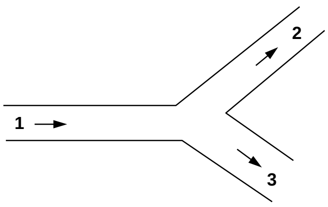
Now try this on a *diffuser,* a pipe which expands or diverges as in the figure below,



If $d_1 = 30mm$ and $d_2 = 40mm$ and the velocity $u_2 = 3.0 m/s$.

What is the velocity entering the diffuser?

Velocities in pipes coming from a junction.



mass flow into the junction = mass flow out

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$$

When incompressible

$$Q_1 = Q_2 + Q_3$$

$$A_1 u_1 = A_2 u_2 + A_3 u_3$$

If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter 40mm takes 30% of total discharge and pipe 3 diameter 60mm.

What are the values of discharge and mean velocity in each pipe?