This question paper consists of 5 printed pages,
(including the formula sheet)
each of which is identified by the Code Number CIVE 140001

Formula Sheet attached

May/June 2005

Examination for the degree of

BEng/ MEng
Civil Engineering

FLUID MECHANICS

Time allowed: 2 hours

Attempt **4** questions

1. **(a)** Starting with the Bernoulli and Continuity equations, show that the following expression gives the discharge measured by a venturimeter.

$$Q = C_d A_1 A_2 \sqrt{\frac{2g\left(\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right)}{A_1^2 - A_2^2}}$$

[7 marks]

(b) A horizontal venturimeter is used to measure the flow of water in a 200mm diameter pipe. The throat diameter of the venturimeter is 80mm and the discharge coefficient is 0.85. If the pressure difference between the two measurement points is 10cm of mercury, calculate the average velocity in the pipe. Assume the relative density of mercury is 13.6.

[8 marks]

(c) The velocity of the water flowing in the same pipe is also measured using a pitot-static tube located centrally in the flow. If the height measured on the attached manometer is 60mm and the relative density of the manometer fluid is 1.45, determine the velocity of the water.

[8 marks]

(d) Explain why the velocity measured by the pitot-static tube is higher than that measured by the venturimeter.

[2 marks]

1.

(a) As per notes

(b) $A_1 = \dfrac{\pi \times 0.2^2}{4} = 0.031416 m^2$

$$A_2 = \frac{\pi \times 0.08^2}{4} = 5.0265 \times 10^{-3} m^2$$

$$P_1 - P_2 = 13.6 \times 1000 \times 9.81 \times 0.1 = 13341.6 Pa$$

$$z_1 - z_2 = 0$$

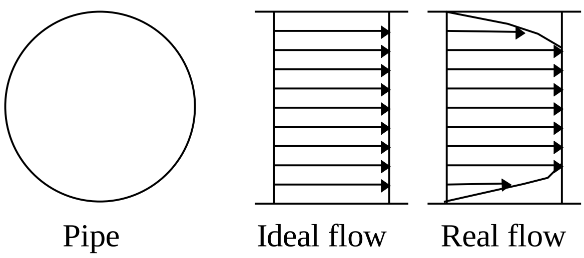
$$Q = 0.85 \times 0.031416 \times 0.0050265 \times \sqrt{\frac{2 \times 9.81 \times \frac{13341.6}{1000 \times 9.81}}{0.031416^2 - 0.0050265^2}} = 0.02238 m^3/s$$

$$v = Q/A_1 = 0.02238/0.031416 = 0.712 m/s$$

(c) $v = \sqrt{\dfrac{2gh(\rho_m - \rho)}{\rho}}$

$$v = \sqrt{\frac{2 \times 9.81 \times 0.06 \times (1450 - 1000)}{1000}} = 7. m/s$$

(d) Velocity measured by pitot-tube is velocity at centre of pipe, while venturi determines average velocity across the whole pipe cross section. This value is lower due to the lower velocities at the walls of the pipe caused by friction.



Velocity profile across a pipe

2. **(a)** Derive the following expression for the discharge over a rectangular weir. Start with Bernoulli's equation and state all the assumptions that you make.

$$Q = C_d \frac{2}{3} B\sqrt{2g} H^{3/2}$$

[7 marks]

(b) Water is flowing over a sharp-crested rectangular weir of width 50cm into a tank with cross-sectional area 0.6 m^2. After a period of 30 s the depth of water in the tank is 1.4m. Assuming a discharge coefficient of 0.9, determine the height of the water above the weir.

[9 marks]

(c) If the rectangular weir is replaced by a 90^o notch weir with the same head and a discharge coefficient of 0.8, calculate the depth increase of the water in the tank after 30s.

[9 marks]

2.

(a) As notes

(b) $Q = \dfrac{0.6 \times 1.4}{30} = 0.028 m^3 / s$

$$H^{3/2} = \frac{Q}{C_d}\frac{3}{2}\frac{1}{B\sqrt{2g}} = \frac{0.028}{0.9} \times \frac{3}{2} \times \frac{1}{0.5 \times \sqrt{2 \times 9.81}} = 0.021071$$

$$H = \sqrt[3]{0.021071^2} = 0.0763m$$

(c)

$$Q = C_d \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right) H^{5/2} = 0.8 \times \frac{8}{15} \times \sqrt{2 \times 9.81} \times 1 \times 0.0763^{5/2} = 0.00304$$

$$height = \frac{30 \times 0.00304}{0.6} = 0.152m$$

The original question said 320° which is not correct.
The rest of the solution has also changed.

3. The discharge of water through a 140° bend, shown in Figure 1, is 30 litres/s. The bend is lying in the horizontal plane and the diameters at the entrance and exit are 200mm and 100mm respectively. The pressure measured at the entrance is 100 kN/m^2, what is the magnitude and direction of the force exerted by the water on the bend?

[22 marks]

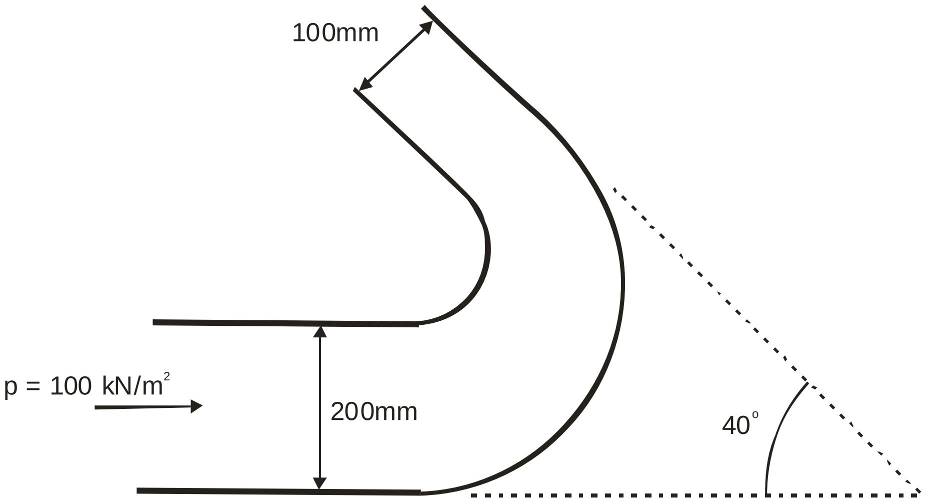


Figure 1

Comment on how frictional losses might be included in the above analysis.

[3 marks]

Solution

$A_1 = \pi d_1 / 4 = 0.0314$ m^2 $\quad$ $A_2 = \pi d_2 / 4 = 0.00785$ m^2 $\quad$ $Q = 30 / 1000 = 0.03$ m^3/s

$u_1 = Q/A_1 = 0.5/0.0314 = 0.955$ m/s $\quad$ $u_2 = Q/A_2 = 0.5/0.00785 = 3.82$ m/s $\quad$ $p_1 = 100$ kN/m^2 = 100 000 N/m^2

Calculate the total force

In the x-direction:

$$\begin{aligned} F_{T\,x} &= \rho Q\left(u_{2x} - u_{1x}\right) = \rho Q\left(u_2 \cos\theta - u_1\right) \\ &= 1000 \times 0.03\left(3.82 \cos 140 - 0.955\right) \\ &= -161.4N \end{aligned}$$

In the y-direction:

$$\begin{aligned} F_{T\,y} &= \rho Q\left(u_{2y} - u_{1y}\right) = \rho Q u_2 \sin\theta \\ &= 1000 \times 0.03 \times 3.82 \sin 140 \\ &= 73.7N \end{aligned}$$

Calculate the pressure force

Use Bernoulli to calculate force at exit, p_2 $\quad$ $\dfrac{p_1}{\rho g} + \dfrac{u_1^2}{2g} + z_1 = \dfrac{p_2}{\rho g} + \dfrac{u_2^2}{2g} + z_2 + h_f$

the friction loss h_f can be ignored, $h_f = 0$

As the exit of the pipe is in the horizontal plane $z_1 = z_2$ so they cancel out.

By continuity, $Q = u_1 A_1 = u_2 A_2$

$$\begin{aligned} p_2 &= p_1 - \frac{\rho}{2}\left(u_2^2 - u_1^2\right) + \left(z_1 - z_2\right)\rho g \\ &= 100000 - \frac{1000}{2}\left(3.82^2 - 0.955^2\right) + 0 \\ &= 93160.0N \end{aligned}$$

$$F_P = \text{pressure force at 1 - pressure force at 2}$$

$$\begin{aligned} F_{P\,x} &= p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos 140 \\ &= 3702N \\ F_{P\,y} &= p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin 50 \\ &= -470N \end{aligned}$$

<u>Calculate the body force</u> (In horizontal plane so no body forces need be considered)

$$F_{Bx} = 0 \qquad\qquad F_{By} = 0$$

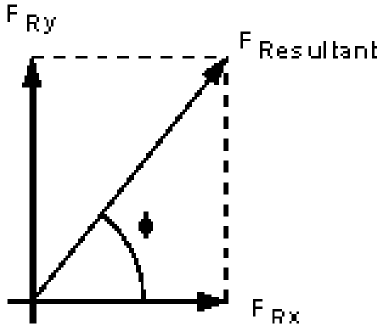
<u>Calculate the resultant force</u>

$$F_{T\,x} = F_{R\,x} + F_{P\,x} + F_{B\,x}$$
$$F_{T\,y} = F_{R\,y} + F_{P\,y} + F_{B\,y}$$

$$F_{R\,x} = F_{T\,x} - F_{P\,x} - F_{B\,x} = -3818N$$

$$F_{R\,y} = F_{T\,y} - F_{P\,y} - F_{B\,y} = 543N$$

And the resultant force on the fluid is given by



$$F_R = \sqrt{F_{R\,x}^2 - F_{R\,y}^2} = 3857N$$

And the direction of application is

$$\phi = \tan^{-1}\left(\frac{F_{R\,y}}{F_{R\,x}}\right) = -8.1^\circ$$

the force on the bend is the same magnitude but in the opposite direction

$$R = -F_R$$

b) The frictional force would be taken into account with a head loss term of the form $h_f = k\dfrac{u^2}{2g}$ in the Bernoulli equation. i.e.

$$\frac{p_1}{\rho g} + \frac{u_1}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2}{2g} + z_2 + h_f$$

4. (a) What is the absolute pressure in the sea at a depth of 10m? Assume the density of seawater is constant at 1025 kg/m^3 and that atmospheric pressure is 101325 Pa.

[3 marks]

(b) If a mercury manometer is attached to a tank of oil or relative density 0.8 and the readings were as shown in Figure 2, what would be the level of water (h) above the point where the manometer was attached?

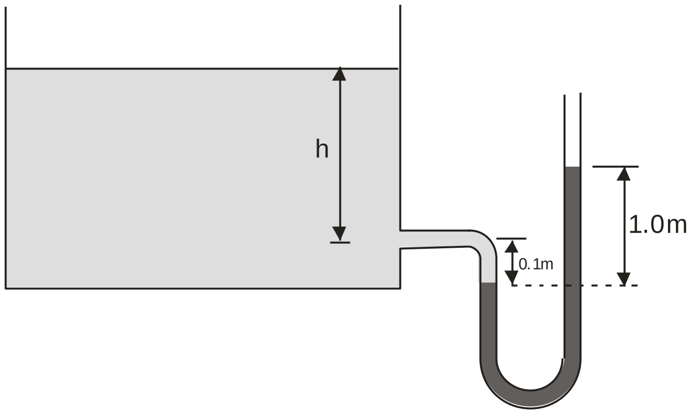


Figure 2

[7 marks]

(c) A storage tank has vertical sides and is filled with water to a depth of 1.5m and this is covered with a 0.5 m depth of oil (of relative density 0.8).
Calculate:
(i) the resultant force on the wall of the tank due to the fluids
(ii) the moment caused by the fluids along wall at the base of the tank.

[9 marks]

(d) In an experiment a jet of water of diameter 10mm is fired vertically upwards at a horizontal sprung target. If a 500g mass placed on the target balances the force of the jet, was is the discharge of the jet in litres/s?

[6 marks]

Solution

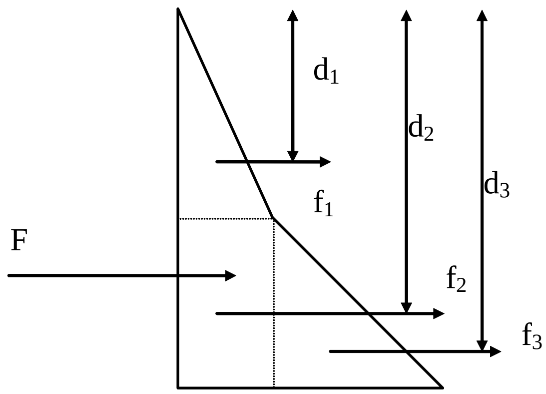
a)

$$p = \rho_{sea} gh + p_{atmospheric}$$
$$p = 1025 \times 9.81 \times 10 + 101325$$
$$p = 201877.5\,Pa \qquad \left(or\ N/m^2\right)$$

b)

$$\rho gh + \rho g 0.1 = \rho_{Hg} g 1.0$$
$$0.8(h + 0.1) = 13.6$$
$$h = 16.9m$$

c)



$$f_1 = \frac{1}{2}\rho_o g h_o^2 = 0.5\times 800\times 9.81\times 0.5^2 = 981\,N$$

$$f_2 = \rho_o g h_o h_w = 800\times 9.81\times 0.5\times 1.5 = 5886\,N$$

$$f_3 = \frac{1}{2}\rho_w g h_w^2 = 0.5\times 1000\times 9.81\times 1.5^2 = 11036.25\,N$$

$$d_1 = \frac{2}{3}h_o = 0.333$$

$$d_2 = h_o + \frac{1}{2}h_w = 1.25$$

$$d_3 = h_o + \frac{2}{3}h_w = 1.5$$

Total force = $F = f_1 + f_2 + f_3 = 17903.25\,N$

Moments about top of surface = $M = f_1 d_1 + f_2 d_2 + f_3 d_3 = 24238.88\,Nm$

Depth to force = $D = \frac{M}{F} 1.354\,m$

Distance from base = 2.0 – 1.354 = 0.646 m

Moment from base = 0.646 * 17903.25 = 11565.5 Nm

d)

$$Mg = \rho Q(v_1 - v_2 \cos 90)$$

$$Mg = \rho Q v_1$$

$$Mg = \rho Q \frac{Q}{A_{jet}}$$

$$Q^2 = \frac{MgA_1}{\rho}$$

$$Q^2 = \frac{0.5\times 9.81\times \pi\times 0.1^2}{4\times 1000} = 3.85\times 10^{-7}$$

$$Q = 0.00062\ m^3/s$$

$$Q = 0.62\,litres/s$$

5. Describe with the aid of diagrams the following:

(i) Newton’s Law of Viscosity

[5 marks]

(ii) The laminar boundary layer

[5 marks]

(iii) The turbulent boundary layer

[5 marks]

(iv) Boundary layer separation

[5 marks]

(v) Methods to prevent boundary layer separation

[5 marks]

(The answer to each part requires at least a half page description of the phenomenon plus diagrams).

6. (a) A 4m diameter circular butterfly gate that pivots about a horizontal axis through its centroid is fitted in the vertical wall of a reservoir as shown in Figure 3.

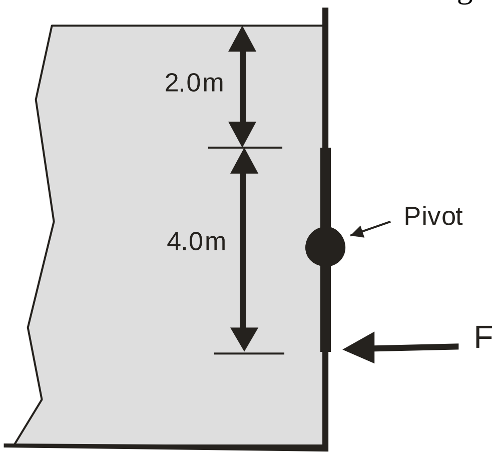


Figure 3

The force due to the water in the reservoir is counteracted by a force F applied at the bottom of the gate to maintain the gate closed. What magnitude of force would be required if the level of water in the reservoir was 2.0 m above the top of the gate? (The second moment of area of a circle is given by $I_{GG} = \pi d^4/64$)

[12 marks]

(b) Figure 4 shows a semicircular gate of 1.2 m diameter and length 0.9m in the wall of a tank containing water. If the gate has a mass of 1000 kg, what force, F, is required to lift the gate?

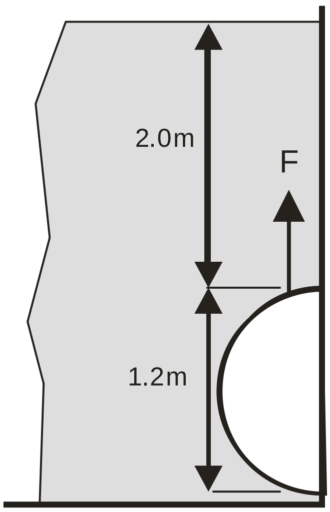


Figure 4

[13 marks]

Resultant force

R = pressure at centroid × area, $R = \rho g(2+2)\frac{\pi 4^2}{4} = 493104\,N$

End of Paper

Depth to centre of pressure, the point where the resultant force acts $Sc = \dfrac{I_{GG}}{A\bar{x}} + \bar{x}$

$$Sc = \frac{12.566}{\frac{\pi 4^4}{4} 4} + \frac{\pi 4^4}{4} = 4.25\, m$$

Moment of R about pivot $M_R = 493104(4.25 - 4.0) = 123276\, Nm$

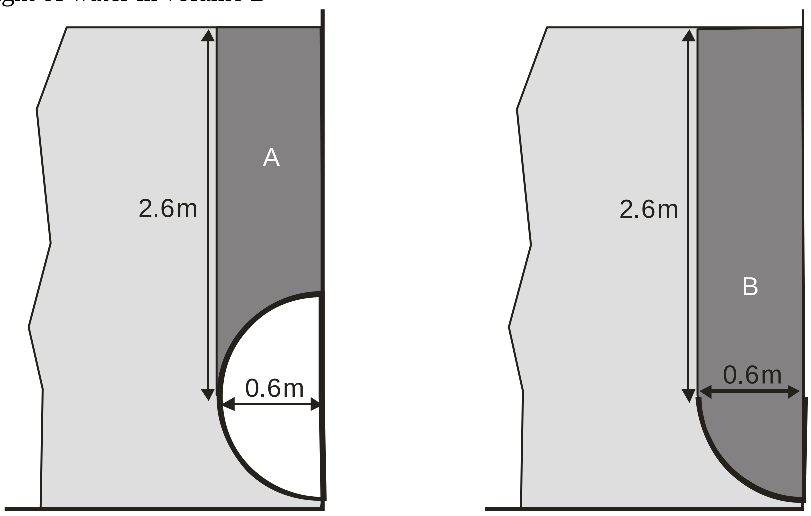
Moment required from F applied at base is M_F is equal to M_R

$$M_F = M_R$$
$$F \times 2 = 123276$$
$$F = 61638\, N$$

b)

Force Down = weight of water above gate (volume A) + weight of gate

Force Up = F + Weight of water in volume B



$$F_D = \left(2.6 \times 0.6 - \frac{\pi 0.6^2}{4}\right) 0.9 \times 1000 \times 9.81 + 1000 \times 9.81 = 21087\, N$$

$$F_U = \left(2.6 \times 0.6 + \frac{\pi 0.6^2}{4}\right) 0.9 \times 1000 \times 9.81 = F + 16270$$

$$F_D = F_U$$
$$21087 = F + 16270$$
$$F = 21087 - 16270 = 4817\, N$$